

Dimensionless Analysis for Regenerator Design

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ABSTRACT

Regenerative heat exchangers represent a crucial component in the design of single and multi-stage cryocoolers. Both the heat transfer and fluid dynamics that occur in the regenerator influence its performance significantly and can only be modeled adequately by solving the coupled mass, energy, and momentum conservation equations. Because of the inherent sophistication required, numerical solutions describing regenerator performance require substantial computational time in order to converge which limits their utility with respect to optimization and design. This report investigates the extrapolation of the performance of a regenerator from a known base case to conditions removed from the base case using a simple, analytical model that is based on running a few simulations in REGENv3.2 at conditions near the base case and correlating the results in terms of the key dimensionless parameters. The analytical model can be used for accelerated parametric study and optimization of a pulse tube cryocooler; the process of fitting an extrapolating function to the REGENv3.2 results must be accomplished periodically as the optimization process moves away from the base case. In this paper, the correlating functions are discussed and the methodology is demonstrated. The base case performance is determined using REGENv3.2 and the extrapolated performance, using the correlating functions, is compared to the performance at the same condition predicted directly by REGENv3.2. Correlations are developed to predict the mass flow rate and phase at the warm end of the regenerator, as well as the heat exchanger ineffectiveness and pressure drop.

INTRODUCTION

The regenerator-design code developed at NIST[1] in its most recent version, REGENv3.2, provides a powerful tool for the user to investigate the influence of geometry, material selection, frequency, temperature, pressure ratio, and the phase between flow and pressure on regenerator performance. While the broad array of allowable input parameters enables a wide variety of questions to be investigated, the multiplicity of choices can be intimidating to the new user, and the optimal approach for utilizing REGENv3.2 for regenerator design is not immediately obvious. Also, the program is computationally intensive to run, particularly for low temperature simulations. A large number of iterations (around 10,000), time steps per cycle (around 250), and spatial mesh points (around 200) are required in order to guarantee the accurate results [2]. These numerical parameters cause REGENv3.2 to require approximately 24 hours to simulate a single condition when run on a typical personal computer. The intent of this investigation is to identify correlating functions that are based on the key dimensionless parameters and can be used to extrapolate the performance of a regenerator over a wide range of operating conditions based

on a minimum number of REGENv3.2 simulations. These correlating functions can subsequently be integrated with models of the other components in a system and used to calculate the regenerator performance within an extensive optimization and design process without having to repeat the detailed investigation with REGEN at every operating condition. As the optimization process moves away from the base case that is used to generate the correlating functions, the extrapolation will be less accurate and, eventually, the process must be repeated using a new base case.

FEATURES OF REGENV3.2

The performance of the regenerator is critical to the overall performance of a pulse tube system because the dominant loss mechanisms in the cycle occur in the regenerator. The input parameters required by REGENv3.2 include the operating parameters (e.g., average pressure, pressure ratio, frequency, etc.) and the geometry (e.g., diameter, length, matrix type, etc.). Within a system simulation, REGENv3.2 is most often used to move from the cold end to the warm end of the regenerator. That is, the amplitude of the mass flow rate at the cold end (\dot{m}_c) and its phase relative to the pressure variation (θ_c) are typically inputs to the simulation while the amplitude of the mass flow rate at the hot end of the regenerator (\dot{m}_h) and its phase angle relative to the pressure variation (θ_h) are typically outputs. Additionally, REGENv3.2 predicts the average pressure drop across the regenerator (ΔP) and the average rate of enthalpy flow through the regenerator ($EHTFLX$); the $EHTFLX$ parameter characterizes the net thermal performance of the regenerator and includes the loss of refrigeration caused by the ineffectiveness of the regenerator as well as heat conduction through the matrix.

The remainder of this paper discusses the correlating functions that have been developed to allow the prediction of these key outputs (\dot{m}_h , θ_h , ΔP , and $EHTFLX$) over a wide range of operating conditions. The mass flow rate and its phase at the hot end are predicted based on a simple, phasor analysis of the regenerator. The enthalpy flux and pressure drop are predicted based on preparing locally valid and well-behaved correlations for the ineffectiveness and friction factor of the regenerator.

HOT END MASS FLOW RATE AND PHASE

The mass flow rate at the hot end and its phase must be estimated before the thermal loss or pressure drop as this quantity feeds the correlating functions for the others. The hot end mass flow rate, \dot{m}_h , and its phase with respect to pressure, θ_h , are predicted approximately using a phasor analysis, as discussed in [3]. The phasor model neglects flow-induced pressure loss through the regenerator and considers only the effect of mass storage in the component. In this limit, the pressure is spatially uniform and can be represented by the average pressure (\bar{P}) and dynamic pressure amplitude (\tilde{P}):

$$P = \bar{P} + \tilde{P} \sin(\omega t) \quad (1)$$

where the pressure amplitude can be expressed as:

$$\tilde{P} = \frac{(PR - 1)}{(PR + 1)} \bar{P} \quad (2)$$

and ω is the angular frequency and PR is the pressure ratio, defined by the ratio of the maximum pressure to the minimum pressure during the cycle. The derivative of pressure with respect to time (t) is given by:

$$\frac{dP}{dt} = \tilde{P} \omega \cos(\omega t) = \tilde{P} \omega \sin\left(\omega t + \frac{\pi}{2}\right) \quad (3)$$

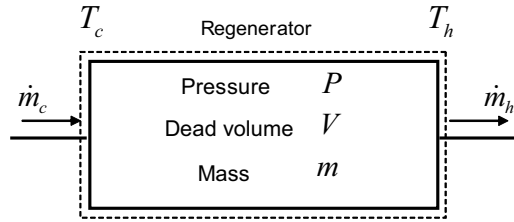


Figure 1. Schematic of the regenerator

Figure 1 illustrates a schematic of the regenerator and shows that an instantaneous mass balance is:

$$\dot{m}_h = \dot{m}_c + \frac{dm}{dt} \tag{4}$$

where m is the mass of gas stored in the regenerator.

The gas stored in the regenerator is assumed to obey the ideal gas law; the mass of the gas is therefore:

$$m = \frac{PV}{RT} \tag{5}$$

where V is the void volume of the regenerator and T is the mass average temperature of the regenerator, which is assumed to be constant. If a linear temperature distribution is assumed, then \bar{T} is [3]:

$$\bar{T} = \frac{T_h - T_c}{\ln\left(\frac{T_h}{T_c}\right)} \tag{6}$$

Substituting Eq. (5) into Eq. (4) yields:

$$\dot{m}_h = \frac{dm}{dt} + \dot{m}_c = \frac{d}{dt} \frac{PV}{R\bar{T}} + \dot{m}_c = \frac{V}{R\bar{T}} \frac{dP}{dt} + \dot{m}_c \tag{7}$$

Note that the mass storage term in Eq. (7) is proportional to the time derivative of pressure; according to Eqs. (1) and (3), the phasor representing mass storage in the regenerator must be oriented at 90° relative to the real axis. The phasor relation expressed by Eq. (7) is shown in Fig. 2 where the magnitude of the mass storage term is

$$\left| \frac{V}{R\bar{T}} \frac{dP}{dt} \right| = \frac{\omega \bar{P} V}{R\bar{T}} \tag{8}$$

In Fig. 2, the mass flow rates are represented as phasors; the magnitude of the phasor represents the amplitude of the mass flow variation while the angle between the phasor and the real axis represents the phase between the mass flow rate and the pressure. A mass flow phasor that lies on the real axis is in phase with the pressure variation. Mass flow phasors that lie in the 1st quadrant lead the pressure variation in time while those in the 4th quadrant lag the pressure variation.

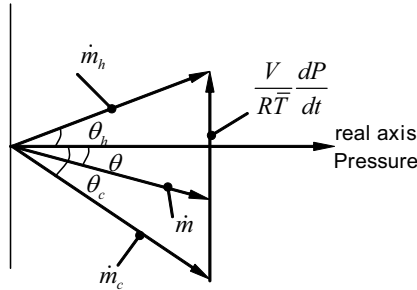


Figure 2. Phasor relationship for the regenerator

In order to explore the limits of the correlating functions discussed in this paper, the function is generated at a particular base case and then used over a range of operating conditions. The result predicted by the correlating function is compared with the result obtained from running REGENv3.2 directly. No base case is needed to accomplish the phasor analysis; however, Table 1 provides the range of conditions used to evaluate its accuracy. Figs. 3(a) and 3(b) show the hot end mass flow rate and phase angle, respectively, predicted using phasor analysis as a function of the same quantity predicted directly using REGEN. Notice that under most operating conditions the phasor analysis agrees with REGEN to within $\pm 15\%$. The larger deviations occur when the pressure drop across the regenerator, which was neglected in the phasor analysis, becomes large. This condition is not typical of a well-defined regenerator.

Table 1. The range of the geometry and operating conditions used to generate Fig. 3

Parameter	Range
Mean pressure (MPa), \bar{P}	2 to 3 MPa
Pressure ratio, PR	1.1 to 1.3
Mass flow rate at the cold end, \dot{m}_c	0.1 to 0.5 kg/s
Phase at the cold end, θ_c	-10 to -80°
Porosity, ϕ	0.5 to 0.8
Frequency, f	30 to 60 Hz
Length, L_r	2 to 13 cm
Area, A_r	100 to 500 cm ²
Matrix type	Screen
Matrix material	Stainless steel

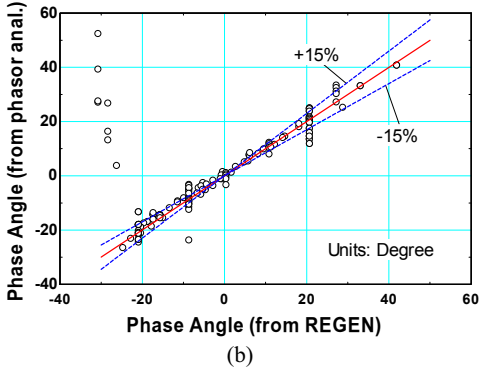
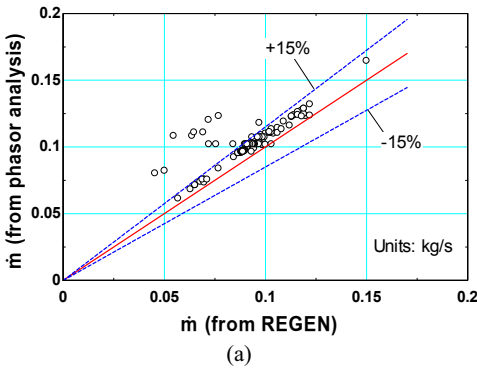


Figure 3. Comparison of (a) the mass flow rate at the hot end of the regenerator predicted by the phasor analysis as a function of the same quantity predicted directly from REGENv3.2 and (b) the phase angle at the hot end of the regenerator predicted by the phasor analysis as a function of the same quantity predicted directly from REGENv3.2

The average mass flow rate within the regenerator can be estimated using the phasor analysis and provides an input to the correlating functions for the thermal and pressure loss. The mass flow rate at any point along the regenerator can be represented as a phasor and the amplitude of the average mass flow rate within the regenerator $|\dot{m}_{avg}|$ is obtained by integrating this phasor in phase space from the cold end to the hot end of the regenerator:

$$|\dot{m}_{avg}| = \frac{1}{\theta_h - \theta_c} \int_{\theta_c}^{\theta_h} |\dot{m}| d\theta \quad (9)$$

where the amplitude of the mass flow rate is a function of phase angle (Fig. 2):

$$|\dot{m}| = \frac{|\dot{m}_c \cos \theta_c|}{\cos \theta} \quad (10)$$

Substituting Eq. (10) into Eq. (9) leads to:

$$|\dot{m}_{avg}| = \frac{1}{\theta_h - \theta_c} \int_{\theta_c}^{\theta_h} \frac{|\dot{m}_c \cos \theta_c|}{\cos \theta} d\theta \quad (11)$$

carrying out the integration leads to:

$$|\dot{m}_{avg}| = \frac{|\dot{m}_c \cos \theta_c|}{\theta_h - \theta_c} (\ln(\sec \theta_h + \tan \theta_h) - \ln(\sec \theta_c + \tan \theta_c)) \quad (12)$$

This average mass flow rate provides a good reference that is used in the subsequent sections.

REGENERATOR THERMAL LOSS

The output *EHTFLX* is the total rate of thermal loss predicted by REGENv3.2 and includes the heat loss caused by the ineffectiveness of the regenerator and the heat conduction through the matrix. This thermal loss must be deducted from the available acoustic power in the pulse tube when calculating the net cooling power. The thermal performance of the regenerator is complex and influence by factors such as the property variation of the matrix and the working fluid, dispersion and conduction, pressurization losses, etc. The value *EHTFLX* clearly cannot be predicted in any way other than the use of a sophisticated numerical model that solves the coupled mass momentum and energy equations. However, over a small region of operating conditions it is expected that the dimensionless thermal performance of a regenerator can be correlated using a small set of dimensionless numbers. The dimensionless parameters cannot be calculated precisely but can be determined approximately using the known inputs; this idea forms the basis of the correlating function for the regenerator thermal loss.

The dimensionless thermal loss is the ineffectiveness, which is defined as the ratio of the thermal loss to the total heat transferred in the matrix (i.e., the energy required bring the average mass flow from T_c to T_h):

$$ineff = \frac{EHTFLX}{|\dot{m}_{avg}| c_p (T_h - T_c)} \quad (13)$$

where c_p is the heat capacity of the working gas, T_h is the temperature of the warm end and T_c is the temperature of the cold end. Note that c_p , as well as the other properties required to implement the correlating functions, are calculated at the mass average temperature, Eq. (6), and mean pressure.

The independent dimensionless numbers that correlate the ineffectiveness are expected to be the capacity ratio (*CR*) and the number of transfer units (*NTU*). The capacity ratio is the ratio of the capacity of the flow through the regenerator to the capacity of the regenerator matrix. The number of transfer units reflects the ratio of the total conductance of the matrix to the capacity of the flow through the regenerator. As calculated below, these definitions are only approximately correct since the properties and other characteristics are only estimates evaluated at the mean temperature and some average operating condition:

$$CR = \frac{|\dot{m}_{avg}| c_p}{L_r A_r \rho_m c_m (1-\phi)} \quad (14)$$

$$NTU = \frac{\bar{k} Nu A_r L_r}{d_h d_h |\dot{m}_{avg}| c_p} \quad (15)$$

where ρ_m is the matrix density, c_m is the matrix heat capacity, ϕ is the matrix porosity, L_r , and A_r are the length and area of the regenerator, respectively, \bar{k} is the conductivity of the gas (evaluated at the average temperature), d_h is the hydraulic diameter of the matrix and Nu is the Nusselt number characterizing heat transfer between the matrix and the gas. The Nusselt number is calculated according to the Reynolds number and Prandtl number according to [4]:

$$Nu = 0.68 Re^{0.6} Pr^{0.33} \quad (16)$$

$$Re = \frac{|\dot{m}_{avg}| d_h}{\bar{\mu} A_r \phi} \quad (17)$$

where Re is the Reynolds number, and Pr is the Prandtl number, $\bar{\mu}$ is the dynamic viscosity at the average temperature. Note that CR and NTU , as defined by Eqs. (14) and (15), can be calculated for an arbitrary set of operating conditions and geometry without running REGENv3.2.

The form of the correlating function that relates the dependent parameter $ineff$ to the independent parameters NTU and CR is:

$$ineff = \exp\left(a + \frac{b + c NTU}{CR NTU}\right) \quad (18)$$

where a , b , and c are undetermined coefficients based on running REGENv3.2 in a controlled fashion for cases that are small perturbations relative to the base case. Because there are 3 unknown coefficients, three cases must be run using REGENv3.2 to determine the coefficients, a , b and c . These cases include the base case as well as two additional cases. Table 2 gives the geometries and the operating conditions used in the base case.

As an example of this procedure, a first set of values for NTU and CR are determined from the geometry and conditions defined in Table 2 for the base case via Eq.s (14) and (15) and the ineffectiveness for the same set of parameters is determined by REGENv3.2. A second set of NTU and CR values are chosen (somewhat arbitrarily) by decreasing NTU by 20% while increasing CR by 40%. Eq.s (14) and (15) are then used to back out values for area A_r and length L_r . These new values along with the other parameters from Table 2 are used as input to REGENv3.2 to calculate the associated ineffectiveness for this second case. A similar method with a third case where now NTU is increased by 20% compared to the base case, and CR is decreased by 40% compared to the base case, provides a third set of A_r and L_r values from which a third value of ineffectiveness is obtained via REGENv3.2. Finally, the three coefficients in Eq. (18) can be determined from these three sets of NTU , CR , and ineffectiveness values.

Table 2. The geometry and operating conditions used in the base case

Parameter	Range
Mean pressure (MPa), \bar{P}	2.5 MPa
Pressure ratio, PR	1.2
Mass flow rate at the cold end, \dot{m}_c	0.135 kg/s
Phase at the cold end, θ_c	-45°
Porosity, ϕ	0.6858
Frequency, f	45 Hz
Length, L_r	2 cm
Area, A_r	300 cm ²
Matrix type	Screen
Matrix material	Stainless steel

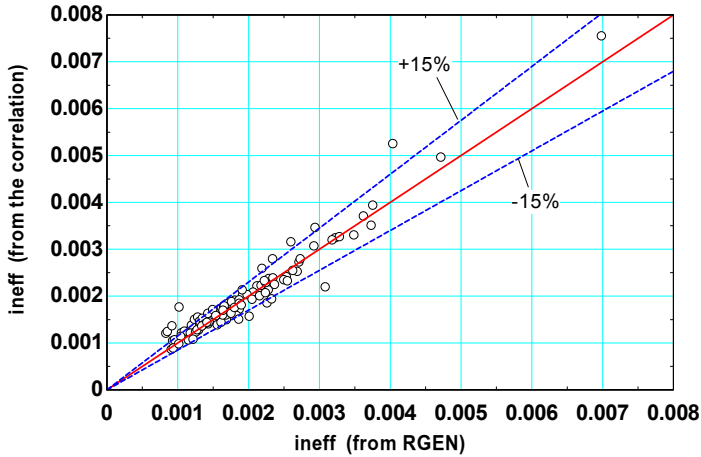


Figure 4 Verify the accuracy of the correlation when the operating conditions change

Additional cases have also been run to verify the accuracy of the correlation with the coefficients given by the base case and the two additional cases. The ranges within which the parameters listed in Table 1 have been varied to test the correlation are also defined in Table 1.. The value of *EHTFLX* for all these cases is determined by *REGENv3.2*, and then converted to the associated value of ineffectiveness via Eq. (13). The values of the ineffectiveness determined in this way define the x-axis in Fig. 4. The ineffectiveness values defining the y axis, are determined via Eq. (18) from their respective vales of *NTU* and *CR* and the previously determined coefficients *a*, *b*, and *c*.. The graph demonstrates that the correlation can be used to predict the ineffectiveness to within 15% of the *REGENv3.2* value for 95% of the cases.

REGENERATOR PRESSURE DROP

The pressure drop across the regenerator can be correlated most conveniently in terms of a friction factor (*f*), which is a dimensionless parameter defined according to:

$$f = \frac{\Delta P d_h \bar{\rho}}{2L_r} \left(\frac{\phi A_r}{|\dot{m}_{avg}|} \right)^2 \tag{19}$$

where $\bar{\rho}$ is the gas density at the average temperature and pressure. The friction factor is assumed a function of Reynolds number, which was defined in Eq. (17). The correlating function that relates the friction factor to the Reynolds number is:

$$f = d + \frac{e}{Re^g} \tag{20}$$

where *d*, *e* and *g* are undetermined coefficients. In a similar fashion as with the ineffectiveness calculation, three cases must be run using *REGENv3.2* to determine the coefficients, *a*, *b* and *c*.in Eq. (20). Again, Table 2 provides the geometries and operating conditions used in the base case. *REGENv3.2* is used to determine the pressure drop for each of the three cases, and Eq. (19) is used to calculate the associated friction factor. With the known friction factors and Reynolds numbers, the coefficients *d*, *e* and *g* are determined by the three specific versions of Eq. (20).

Additional cases have also been run to verify the accuracy of the correlation as compared to the precise solution from *REGENv3.2*.. The ranges within which the parameters listed in Table 1 have been varied to test the correlation are also defined in Table 1. The comparison is shown in Fig. 5 where the pressure drop defining the x-axis is determined by *REGENv3.2*, while that defining the y-axis is determined from Eq.s (20) and (19) for the predetermined values of *d*, *e*, and *g*.. The graph demonstrates that the correlation predicts the friction factor to within 15% of

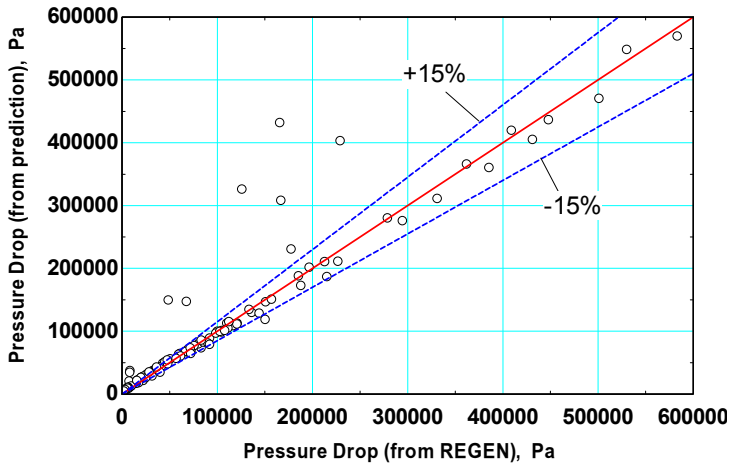


Figure 5 Pressure drop from REGEN vs. pressure drop from the prediction of the correlations

the value provided by REGENv3.2. The large disagreement occurs when the porosity is different than the base case.

DISCUSSION AND CONCLUSIONS

REGENv3.2 provides a powerful and accurate numerical model for regenerator design. However, in some cases an excessive amount of time is required to obtain useful results. An alternative method, utilizing a combination of phasor analysis and dimensionless correlations can be used to greatly shorten the calculation time for regenerator designs. The phasor analysis is used to calculate the mass flow rate and phase with respect to pressure at the warm end, and predicts these to within 15% of the values precisely determined by REGENv3.2. Correlations have also been developed to determine the ineffectiveness and pressure drop losses of the regenerator. The correlations require running three known cases using REGENv3.2 to determine the coefficients for either the ineffectiveness or the pressure drop. Once the coefficients are determined, the correlations can be used to estimate the ineffectiveness or pressure drop to within 15% of the values precisely determined by REGENv3.2. Furthermore, the correlations are useful over a fairly large range in a variety of parameters including the average pressure, pressure ratio, frequency, porosity, and cold end mass flow rate, phase angle, and temperature.

From the above discussion, we may conclude that the phasor analysis can be used as an accurate mechanism to calculate the mass flow rate and phase at the warm end of the regenerator. The dimensionless correlations can also be used to calculate the thermal loss and pressure drop across the regenerator. Once the correlations are known, they can also be used for design calculations. The primary advantage of this method is that it will shorten the required calculating time for design studies as compared with those required by the full use of REGENv3.2, and yet maintain an acceptable accuracy.

ACKNOWLEDGMENT

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