Effect of Geometric Configuration on the Dynamic Behavior of Linear Compressor and Cooling Performance in Stirling-Type Pulse Tube Refrigerator

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ABSTRACT

For a given linear compressor, the geometric configuration of the PTR (pulse tube refrigerator) affects the dynamic behavior of the linear compressor as well as the cooling performance of the PTR. The geometric configuration of each component precisely determines the flow impedance which contains the flow resistance, the inertance effect, and the compliance effect. In this paper, several simulation sets are performed with an analysis code in consideration of the dynamics of the linear compressor and the thermal losses. For the same magnitude of input current, the swept volume and the resonant frequency of a linear compressor vary with the geometric configuration of the PTR; this is because the flow impedance determines the amount of the gas spring effect and the gas damping effect to the piston. For a linear compressor, a larger swept volume generally results in a higher acceptance of the input power and a better compression efficiency. However, a higher compression power and compression efficiency for a linear compressor do not always guarantee a better cooling performance of the PTR. The simulation results clearly show the existence of the optimum geometric configuration of the PTR for a given linear compressor. The underlying physical reasons are explained with the power transfer from the input power to the cooling capacity.

INTRODUCTION

In a Stirling-type PTR, the PTR is directly connected to a linear compressor which generates the pulsating pressure and the oscillating flow by the linear motion of a piston. The dynamic behavior of a linear compressor is determined by its mechanical characteristics and also the generated pressure wave in the compression space.

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performance of the PTR are affected by the dynamic behavior of a linear compressor. This mutual interaction varies according to the geometric configuration and the operating condition of PTR for the given linear compressor. The performance analysis in this paper considers the dynamic and electric characteristics of a linear compressor, while the previous research adopted the assumed piston displacement or the pressure wave in the compression space[3, 4].

In this paper, the effect of geometric configuration of PTR on the dynamic behavior and the cooling performance of a Stirling-type PTR is investigated with thermo-hydraulic simulation results. First, the effect of inertance tube, which plays an important role in controlling the phase between the pressure and the mass flow rate at the cold-end, is investigated. Especially, the power transfer characteristic from the input electric power to the cooling performance is deeply discussed. Second, the effect of pulse tube and regenerator size is subsequently investigated.

NUMERICAL SIMULATION

In the previous research, we developed the performance analysis code for a Stirling-type PTR with consideration of the dynamics of a linear compressor[5]. In a Stirling-type PTR, all physical variables of the input voltage, the piston displacement, the pressure at each point and the mass flow rate at each interface have nearly sinusoidal waveform for the sinusoidal input current because the nonlinearity of the system is not strong. They are expressed with the amplitude and the phase shift with respect to the waveform of input current.

$$i(t) = I_0 \cos \omega t \tag{1}$$

$$a(t) = A_0 \cos(\omega t + \phi_a) \tag{2}$$

Linear compressor

Eq. (3) is the dynamic motion equation of a piston. The pressure difference between both sides of a piston imposes additional stiffness to the vibration system and the compression PV power in the compression space can be regarded as the damping effect because the energy is transferred out of the vibration system. In Eq. (3), those effects of gas force are represented with the gas spring constant and the gas damping coefficient which vary with the configuration and operating condition of the PTR.

$$m_p \frac{d^2x}{dt^2} = -k_m x - c_m \frac{dx}{dt} - P_c A_p + K_E \cdot i = -(k_m + k_g)x - (c_m + c_g)\frac{dx}{dt} + K_E \cdot i \tag{3}$$

Eq. (4) is derived from mass and energy conservation in the compression space. The piston movement generates pressure wave and an oscillating flow. If the outlet of the compression space is closed, the mass flow rate is zero and the piston displacement and the pressure wave

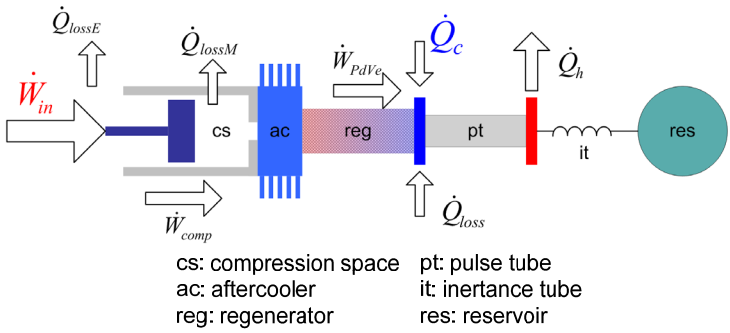


Figure 1. Power transfer in Stirling-type pulse tube refrigerator.

Table 1. Specifications and operating conditions.

Linear compressor		Pulse tube refrigerator	
Moving mass (m_p)	0.4506 kg	Regenerator	3/4 inch (O.D.), 0.3 mm (t)
Mechanical stiffness (k_m)	2687 N/m	Pulse tube	3/8 inch (O.D.), 0.3 mm (t)
Mechanical damping coeff. (c_m)	7 ~ 50 N-s/m	Inertance tube	1/8 inch (O.D.), 0.75 mm (t)
Piston diameter (D_p)	27 mm	Reservoir	350 cc
Maximum displacement amplitude (L_0)	13 mm	**Operating condition**	
		Warm-end temperature	300 K
Motor constant (K_E)	7.6142 N/A	Cold-end temperature	100 K
		Charging pressure	15 atm
		Input current	8.5 Arms

have exactly in-phase relation. On the other hand, if the flow impedance is zero, the pressure wave is not generated and the mass flow rate is simply proportional to the velocity of a piston.

$$\frac{P_m A_p}{RT_{ca}}\frac{dx}{dt} = \frac{A_p L_0}{\gamma R T_{ca}}\frac{dP_c}{dt} + \dot{m}_{ca} \tag{4}$$

Power transfer

Figure 1 shows the power transfer from the input electric power to the cooling capacity in a Stirling-type PTR. The compressor accepts an input electric power and produces a compression PV power except the compression loss in the compression space. The major components of the compression loss are the Joule heating of a coil, the eddy current loss in an iron core and the mechanical friction loss. The generated compression PV power is transferred to the expansion PV power in the cold-end and the remainder except the thermal loss becomes the cooling capacity. The thermal loss mainly consists of the ineffectiveness loss of the regenerator and the shuttle heat loss in the pulse tube. The ineffectiveness loss and the shuttle heat loss are caused by a finite heat transfer characteristic between the gas flow and the regenerator material in the regenerator and the shuttle heat transfer between the gas flow and the wall in the pulse tube, respectively.

The amount of each power is calculated with Eq. (5) ~ (8). The important thing is that the amount of each power is determined by the amplitude as well as the phase difference. For example, when the pressure in the compression space and the piston displacement are in-phase, the compression PV power is zero despite the pressure amplitude and the piston movement. This case appears when the outlet of the compression space is closed. For the pulse tube, if the phase difference between the pressure and the mass flow rate at the cold-end is 90°, the expansion PV power is zero and the refrigeration effect is not generated for the ideal case. A basic PTR shows this phenomenon and its refrigeration effect is generated only by surface heat pumping in it.

$$\dot{W}_{in} = f\oint(v\cdot i)\,dt = \frac{1}{2}V_0 I_0 \cos\phi_v \tag{5}$$

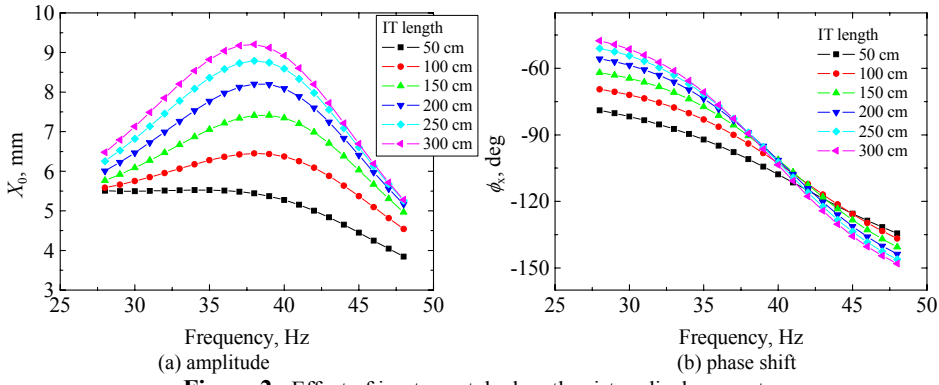
$$\dot{W}_{comp} = -f\oint P_c dV_c = \pi f A_p P_{c0} X_0 \sin\left(\phi_{pc} - \phi_x\right) \tag{6}$$

$$\dot{W}_{PdVe} = f\oint P_e dV_e \cong \frac{f}{\rho_m}\oint P_e \dot{m}_{le} dt = \frac{1}{2\rho_m} P_{e0}\dot{M}_{le0}\cos\left(\phi_{pe} - \phi_{mle}\right) \tag{7}$$

$$\dot{Q}_{ref} = \dot{W}_{PdVe} - \dot{Q}_{loss} \cong \dot{W}_{PdVe} - \left(\dot{Q}_{ineff} + \dot{Q}_{shuttle}\right) \tag{8}$$

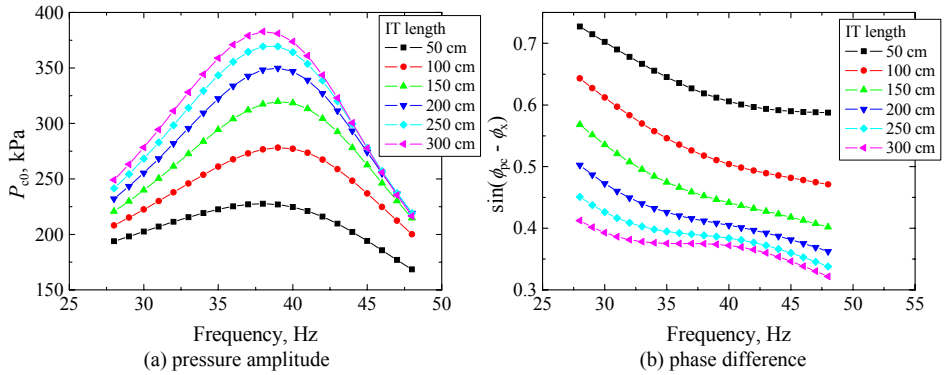
Simulation model

The specifications of the simulation model and the operating conditions are given in Table 1. The specifications of the linear compressor were obtained from the measurement of the actual compressor. The only difference is the range of the piston displacement. In a real compressor, the piston displacement amplitude is limited to 5.5 mm due to its interior structure, but in this



(a) amplitude (b) phase shift

Figure 2. Effect of inertance tube length; piston displacement.



(a) pressure amplitude (b) phase difference

Figure 3. Effect of inertance tube length; pressure wave

simulation, it is not limited up to the neutral length of the cylinder. The input condition is limited in the current intensity due to the heat dissipation through the compressor body. In the simulation, the current intensity is fixed to 8.5 Arms.

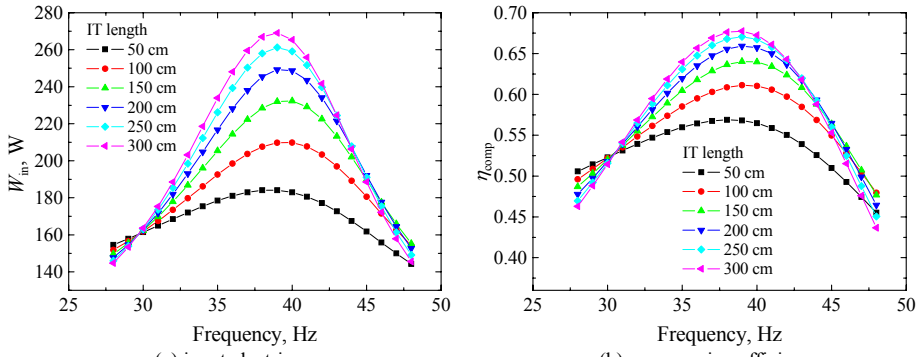
RESULTS AND DISCUSSION

Effect of inertance tube length

The length of the regenerator and the pulse tube are fixed to 6 cm and 8 cm, respectively. The simulation results are obtained for the inertance tube length of 50, 100, 150, 200, 250, 300 cm. Figure 2 shows the frequency response of the piston displacement. As the inertance tube becomes longer, the amplitude at the resonant region increases and the phase shift becomes steeper. This behavior is similar to the behavior influenced by the decreasing damping ratio in the vibration system of mass-spring-damper.

Figure 3 shows the simulation result of the pressure wave. The larger pressure amplitude is naturally generated due to the larger piston displacement while the phase difference between the pressure and the displacement becomes smaller with the longer inertance tube. The longer inertance tube actually increases the flow impedance of the PTR and thus, the piston movement contributes more to the pulsating pressure than the oscillating flow.

Figure 4 shows the power transfer in the linear compressor. The PTR with the longer inertance tube accepts larger amount of input electric power and generates higher compression PV power. This is because that the effect of increasing the amplitude of the pressure and



(a) input electric power (b) compression efficiency

Figure 4. Effect of inertance tube length; power transfer in linear compressor.

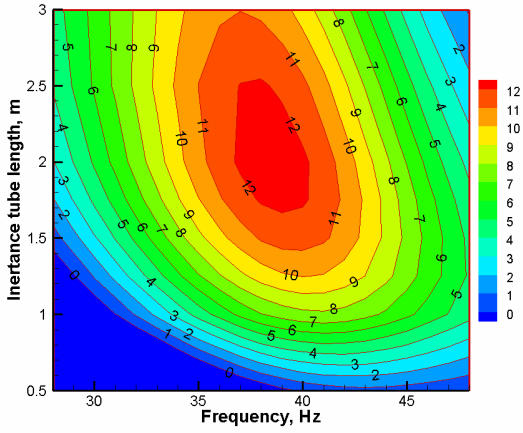
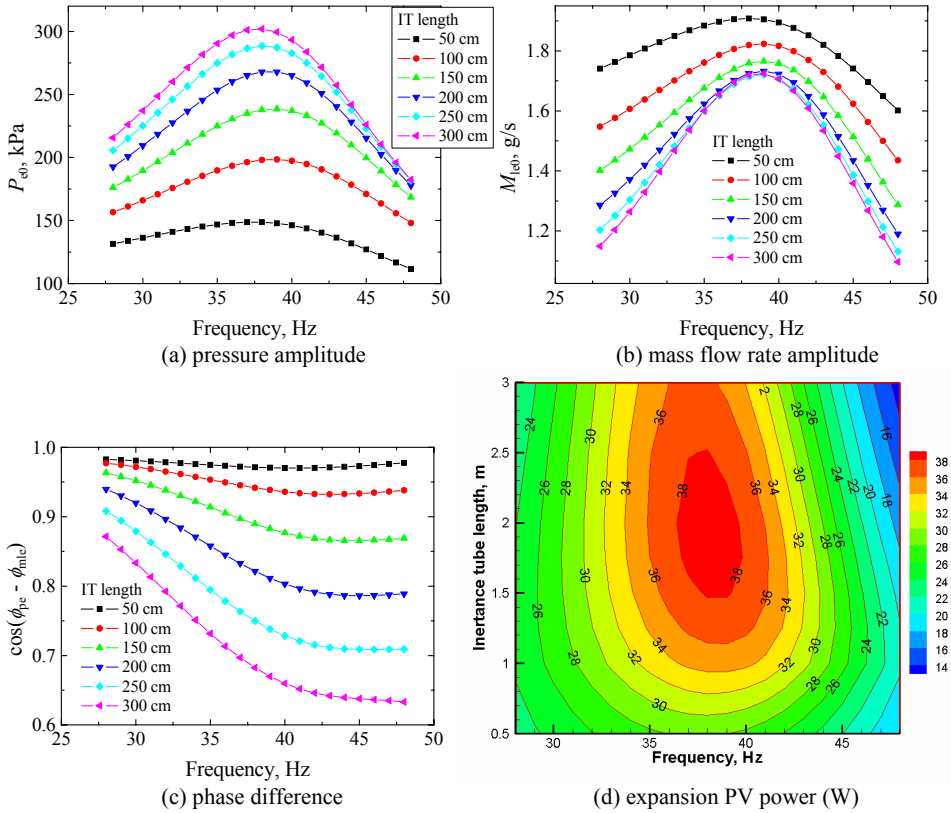


Figure 5. Effect of inertance tube length; cooling capacity (W).

displacement is more dominant than that of decreasing the phase difference between the pressure and displacement. The linear compressor is more efficient with the longer inertance tube but, the higher compression PV power and efficiency of the linear compressor do not guarantee the higher cooling performance of the PTR. The cooling capacity is optimized at a certain inertance tube length as shown in Figure 5. The reason for this behavior can be explained with the PV power transfer to the expansion space and the thermal loss.

Figure 6 shows the behavior of the pressure, the mass flow rate and the expansion PV power in the expansion space. For longer inertance tube, the pressure amplitude increases, while the amplitude of the mass flow rate slightly decreases. The phase difference between the pressure and the mass flow rate increases. As a result, the expansion PV power shows the optimum value with a certain inertance tube length. With the longer inertance tube, the compliance effect of the pulse tube becomes larger because the flow impedance of the inertance tube increases. If the inertance tube length is infinite, the flow impedance is also infinite and the PTR is identical with the basic PTR which has the phase difference of 90° between the pressure and the mass flow rate.

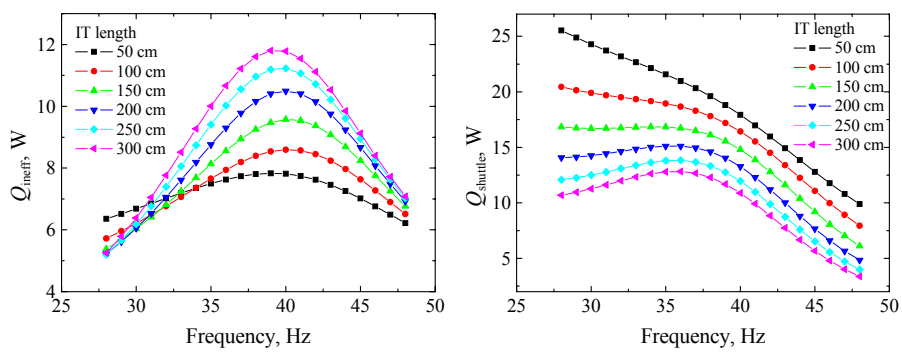
Figure 7 shows the calculation results of the thermal loss. As the inertance tube becomes longer, the ineffectiveness loss increases because the average mass flow rate through the regenerator increases. On the other hand, the shuttle heat loss decreases due to the larger pressure amplitude and the smaller mass flow rate. As a result of the conflicting behavior, the overall thermal loss is slightly affected by the inertance tube length at the resonant region and the variation of the cooling capacity shows the similar behavior of the expansion PV power as shown in Figure 5.



(a) pressure amplitude (b) mass flow rate amplitude

(c) phase difference (d) expansion PV power (W)

Figure 6. Effect of inertance tube length; in the expansion space (cold-end).



(a) ineffectiveness loss (b) shuttle heat loss

Figure 7. Effect of inertance tube length; thermal loss.

Effect of pulse tube and regenerator length

The dynamic behavior of linear compressor and the cooling performance of the PTR are investigated with the simulation results for the various sizes of the regenerator and the pulse tube. Table 2 shows the maximum piston displacement and the resonant frequency for each dimension of the PTR. As the size of pulse tube and regenerator decreases, the resonant frequency becomes

Table 2. Results of the piston displacement amplitude and the resonant frequency.

L_{reg}	L_{pt}	L_{it} = 50		L_{it} = 100		L_{it} = 150		L_{it} = 200		L_{it} = 250		L_{it} = 300	
		X_0	f_0	X_0	f_0	X_0	f_0	X_0	f_0	X_0	f_0	X_0	f_0
6	4	5.22	35	6.20	38	7.31	39	8.26	39.5	8.97	39.5	9.47	39.5
	8	5.52	34	6.45	37	7.41	38	8.20	38	8.79	38	9.21	38
	12	5.79	33.5	6.63	36	7.48	37	8.15	37	8.63	37	8.99	37
	16	6.03	33	6.78	35	7.51	36	8.10	36	8.53	36	8.83	36
3	8	5.28	32	5.65	37.5	6.78	39.5	7.86	40	8.73	40.5	9.40	40.5
6		5.52	34	6.45	37	7.41	38	8.20	38	8.79	38	9.21	38
9		6.11	35	7.09	36.5	7.90	37	8.50	37	8.92	36.5	9.20	36.5
12		6.71	35.5	7.62	36	8.29	36	8.75	35.5	9.05	35	9.25	35

Table 3. Summary on the size effect of pulse tube and regenerator.

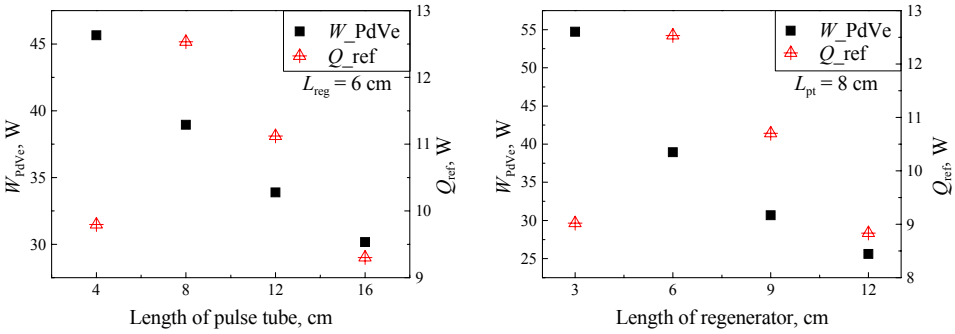
	Decreasing L_{pt} with fixed L_{reg}	Decreasing L_{reg} with fixed L_{pt}
P_{20}	Increase	Increase
$\dot{M}_{le0}$	Decrease	Increase
$\cos\left(\phi_{p2}-\phi_{mle}\right)$	Increase	Decrease
$\dot{W}_{PdVe}$	Increase	Increase
$\dot{Q}_{ineff}$	Decrease	Dramatic increase
$\dot{Q}_{shuttle}$	Dramatic increase	Increase
$\dot{Q}_{ref}$	Optimum value	Optimum value

higher and the displacement amplitude is more sensitive to the variation of the inertance tube length. This is because the flow information of the downstream is communicated to the piston more rapidly.

In Figure 8, the maximum values of the expansion PV power and the cooling capacity show similar tendency for the pulse tube length and regenerator length. The expansion PV power gradually decreases as the size of the pulse tube or the regenerator increases and the cooling capacity shows an optimum value at a specific size. The physical reasons are different for two cases as shown in Table 3.

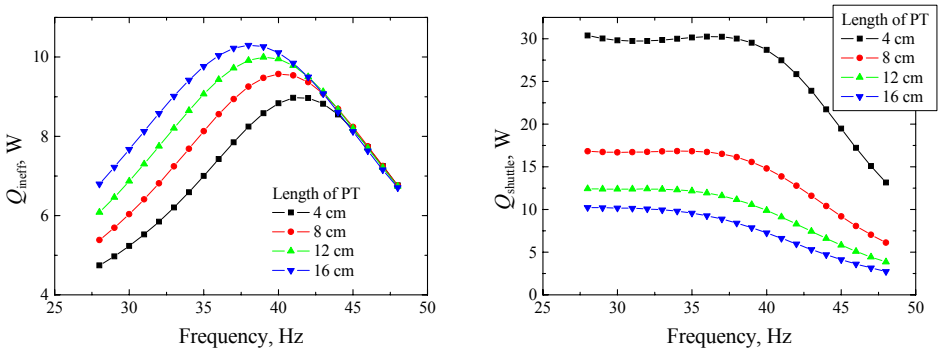
For the case of varying pulse tube size, a smaller pulse tube has smaller compliance and thus, larger pressure amplitude can be obtained with smaller mass flow rate. And also, the phase difference between the pressure and the mass flow rate is small. The expansion PV power, which is calculated by Eq. (7), subsequently increases because the increase of the pressure amplitude and the decrease of the phase difference make more dominant effect than the decrease of the mass flow rate amplitude. The ineffectiveness loss decreases because the average mass flow rate through the regenerator decreases for the smaller pulse tube. However, the shuttle heat loss dramatically increases as shown in Figure 9 due to the increase of the temperature gradient along the pulse tube wall.

For the case of varying regenerator size, a smaller regenerator decreases the compliance of the regenerator and the PV power loss due to the pressure drop. The larger mass flow rate results in the larger pressure amplitude and the larger phase difference between the pressure and the mass flow rate in the expansion space. In this case, the increase of pressure amplitude and mass flow rate amplitude make more dominant effect than the increase of phase difference. This results in the increase of expansion PV power with the smaller regenerator. The ineffectiveness loss and the shuttle heat loss simultaneously increase as the decrease of regenerator size. Especially, the ineffectiveness loss dramatically increases because the mass flow rate through the regenerator increase and the heat transfer area decreases.



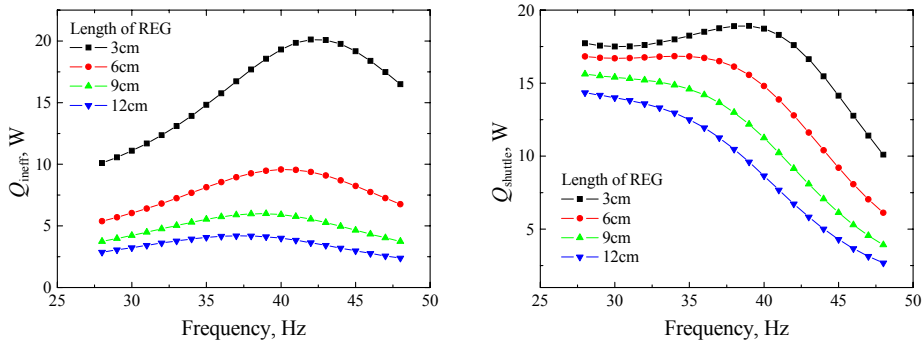
(a) effect of pulse tube length (b) effect of regenerator length

Figure 8. Maximum values of expansion PV power and cooling capacity.



(a) ineffectiveness loss (b) shuttle heat loss

Figure 9. Examples of thermal loss behavior as pulse tube length (L_{it} = 150 cm).



(a) ineffectiveness loss (b) shuttle heat loss

Figure 10. Examples of thermal loss behavior as regenerator length (L_{it} = 150 cm).

CONCLUSIONS

In this paper, we investigated the effect of the inertance tube, the pulse tube and the regenerator on the dynamics of a linear compressor and the cooling performance of a Stirling-type PTR for a given linear compressor. From the simulation results, the followings can be concluded.

- For a given linear compressor and the fixed dimensions of the pulse tube and the regenerator, the longer inertance tube decreases the damping ratio in the vibration system of the piston.
- With the fixed dimensions of the pulse tube and the regenerator, the longer inertance tube results in the higher efficiency of the linear compressor. But, it does not always guarantee the better cooling performance.
- For the same size of pulse tube, the pressure amplitude and the phase difference between pressure and mass flow rate show the conflicting effect on the expansion PV power as the variation of inertance tube length. The optimum length of inertance tube should be selected for efficient power transfer from input electric power to expansion PV power.
- The size effect of the pulse tube and the regenerator show the similar resultant behavior in the expansion PV power and the cooling performance, but the physical reasons are different for two cases.

NOMENCLATURE

v	Voltage [V]	*Greek symbols*	
x	Piston displacement [m]	η	Efficiency
i	Current [A]	ρ	Density
m_p	Moving mass [kg]	ω	Angular velocity
k_m	Mechanical spring constant [N/m]	*Subscripts*	
c_m	Mechanical damping coefficient [N-s/m]	ca	Interface between compression space and aftercooler
K_E	Motor constant [N/A]	le	Interface between cold-end heat exchanger and pulse tube
D_P	Piston diameter [m]	0	Amplitude
A_P	Piston area [m^2]	c	Compression space
L_0	Neutral length of cylinder [m]	e	Expansion space
f	Frequency [Hz]	in	Input power
f_0	Resonant frequency [Hz]	comp	Compression PV power
P	Pressure [kPa]	PdVe	Expansion PV power
T	Temperature [K]	ref	Cooling capacity
$\dot{W}$	Work rate [W]	loss	Thermal loss
$\dot{Q}$	Heat rate [W]	ineff	Ineffectiveness loss of regenerator
$\dot{m}$	Mass flow rate [kg/s]	shuttle	Shuttle heat loss

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