The Role of Thermoconductivity in Pulse Tube Cryocoolers

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ABSTRACT

Descriptions of the thermodynamics of pulse tube cryocoolers usually concentrate on the roles of entropy, enthalpy, pressure, and mass flow. Thermoconductivity is often considered as a parasitic loss. This paper reviews the role thermoconductivity plays in determining not only the temperature profiles in the regenerator and pulse tube but also the interaction between thermal conductivity and real gas effects and in determining the behavior at the transitions between components.

NOMENCLATURE

Roman

A	Area
C_P	Specific heat at constant pressure
E	Energy
$\dot{H}$	Enthalpy flow
k	Thermal conductivity
$\dot{m}$	Mass flow
P	Pressure
$\dot{q}$	Heat flow
$\dot{Q}$	Heat flow
$\dot{S}$	Entropy flow
T	Temperature
t	Time
W	Work
x	General parameter
z	Position

Greek

η	Enthalpy coefficient
μ	Joule-Thomson coefficient
ϕ, φ, θ	Phase angle
ω	Frequency

Subscript

0	Steady term
1	Oscillating term
a	Aftercooler
c	Cold
off	Effective
gen	Generation
h	Hot
i	Index
k	Conduction

INTRODUCTION

A simple one-dimensional steady-state model based on the small amplitude assumption has been a powerful tool in describing the top-level thermodynamics of pulse tube cryocoolers. This model starts with the first two laws of thermodynamics applied to control volumes that includes the working fluid only:

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energy: $$E = \sum \dot{Q}_i - \sum W_i + \sum \dot{H}_i = 0, \quad (1)$$

and entropy: $$S_{gen} = -\sum \dot{Q}_i/T_i - \sum \dot{S}_i, \quad (2)$$

where $\dot{Q}_i$, $\dot{H}_i$, and $\dot{S}_i$ are cyclic average flows into the control volume (Fig. 1) and W_i is the cyclic average work done by the control volume. For a differential control volume where no work is done, Eqs. (1-2) reduce to

$$\nabla \dot{Q} + \nabla \dot{H} = 0 \quad (3)$$

and $$\nabla\left(\dot{Q}/T\right) + \nabla \dot{S} + \nabla S_{gen} = 0. \quad (4)$$

The small-amplitude steady-state approximation assumes that all time dependent quantities are of the form:

$$x = x_0 + x_1 \sin(\omega t + \phi_x) \text{ and } x_0 = \langle x \rangle, \quad (5)$$

where x_0 and x_1 are functions of position only, higher order terms are negligible, and only quantities averaged over a cycle are of interest.

The components of a pulse tube along with the entropy and enthalpy flows for an ideal cryocooler are shown in Fig. 2. The ideal case assumes no viscous losses, no pressure gradients (except in the orifice), and no heat transfer losses. Interactions with the regenerator, heat exchangers, and physical walls are limited to heat and work transfer. In this simple model, the temperature gradients within the regenerator and pulse tube are unspecified; although, the temperatures of all the heat exchangers are specified. The internal heat flow due to thermal conduction is not explicitly included. Instead, thermal conduction is treated as a parasitic external load. This approach has been applied to ideal, non-ideal pulse tubes, and real gas effects of ^{4}He and ^{3}He in ideal pulse tube cryocoolers.[1-4] For real gases, thermal conduction was shown to be necessary for the operation of the regenerator.[3]

(a) (b)

Figure 1. (a) Schematic representation of a generalized control volume in an open system. The quantities m, E, and S can accumulate within the control volume. The nominal directions of the in- and outflows are shown. Heat flows are always associated with an external temperature. (b) Schematic representation of a differential control volume in an open system.

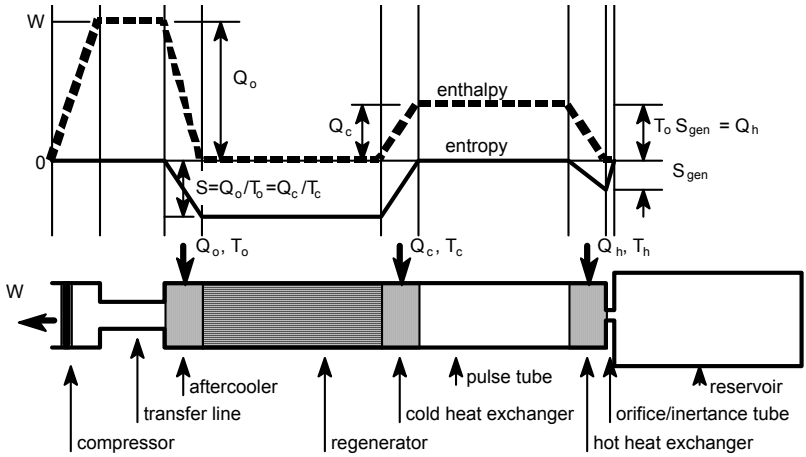

Figure 2. A diagram illustrating the principle components of an idealized pulse tube with the heat, enthalpy, and entropy flows in the various components.

Here, we discuss qualitatively the role thermal conductivity plays in the regenerator and the in the pulse tube. Previous uses of this model have ignored end effects, the transitions between components.[1-4] These effects are important. We discuss the thermal conduction effects with and without the end effects. The basis for the discussion is an ideal pulse tube in the small-amplitude steady-state approximation. Non-ideal effects are included only as needed.

PULSE TUBE TEMPERATURE PROFILE

The pulse tube contains an oscillating, compressible, sub-sonic gas with porous heat exchangers at the ends. In 0-g or when gravity is oriented along the axis, the pulse tube can be modeled two-dimensionally. The flow is primarily axial with a small secondary flow. External heat transfer occurs primarily at the two heat exchangers. The heat transfer to the wall is small (except for the basic pulse tube, which is not considered here).

Ideal case

In the ideal case, the secondary flow and the interactions with the wall are ignored. This leads to a one-dimensional axial model. The thermodynamics reduces to constant enthalpy flow ($\nabla \dot{H} = 0$) and no entropy flow ($\dot{S} = 0$). This description defines the gross cooling power at the cold heat exchanger and the gross heat rejected at the hot heat exchanger:

$$\dot{Q}_c = \dot{Q}_h. \tag{6}$$

This approach of considering only external heat flows does not define the temperature gradient. Rather one must include an internal process, thermal conductivity, to define the temperature profile:

$$\dot{Q}_k = -kA\nabla T, \tag{7}$$

Applying Eqs. (3, 7) to a differential control volume results in:

$$\nabla \dot{H} = -\nabla \dot{Q}_k. \tag{8}$$

The enthalpy and heat flows could vary along the pulse tube. However, this would require a mechanism that couples the temperature amplitude ($T_1 \propto \dot{H}$) with the mean temperature gradient ($\square T_0 \propto \dot{Q}$). This only occurs during heat transfer, such as with the wall or with the heat exchangers at the ends. The heat transfer to the wall is negligible. Thus, away from the ends, $\nabla \dot{H} = -\nabla \dot{Q}_k = 0$.

Since, the thermal conductivity of helium is k ≈ 0.35 $T^{2/3}$ mW/m·K, the mean temperature gradient is

$$\nabla T_0 \propto k^{-1} \propto T_0^{-2/3}. \tag{9}$$

The pressure oscillations cause the temperature to oscillate about T_0 with amplitude T_1.

Pulse tube to heat exchanger transition

This transition is between an isothermal heat exchanger ($\dot{H} = 0$) and the adiabatic pulse tube ($\dot{S} = 0$). At these transitions (see Fig. 2), the enthalpy flow in the pulse tube vanishes. Two-dimensional modeling using a commercial computational fluid dynamics, CFD, application has shown that in this region there are four types of temperature profiles during each cycle.[5] These are shown in Fig. 3 for the transition at the hot heat exchanger. The figures would be inverted for the cold transition. The figures show two types of processes:

1) One occurs when the mass flow is from the heat exchanger and into the pulse tube. The flow leaves the heat exchanger at the temperature of the heat exchanger with no convective heat transfer. Once in the pulse tube, the changing pressure causes the gas temperature to change. This results in a temperature gradient in the pulse tube and thermal conduction between the gas in the pulse tube and the heat exchanger. This process is described by Eq. (7).

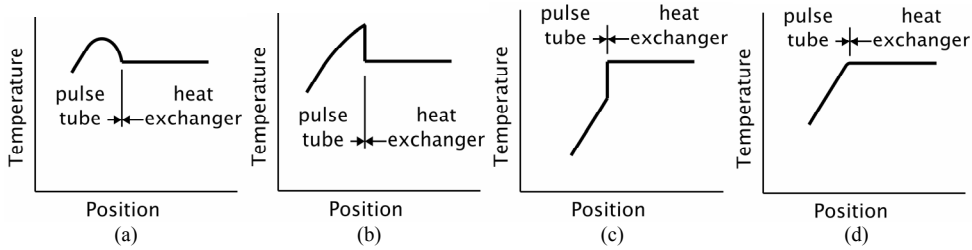

Figure 3. The temperature profile at the pulse tube/hot heat exchanger interface in a pulse tube cryocooler with inertance tube is illustrated in sequence during the cycle. The pressure is above the mean pressure in (a) and (b) and below the mean pressure in (c) and (d). The mass flow is from the heat exchanger into the pulse tube in (b) and (c) and is reversed in (a) and (d).

Table 1. Phase shift used in Fig. 4 between mass flow and pressure (mass flow lag < 0, lead > 0)

Type of Pulse Tube Cooler	at Cold Heat Exchanger	at Hot Heat Exchanger
Orifice	30°	0°
Inertance Tube	-30°	-60°

2) The other occurs when the gas flows from the pulse tube and into the heat exchanger. The gas enters the heat exchanger and its temperature rapidly changes to equilibrate with the heat exchanger. The temperature gradient is very much greater than in the previous case. Thus, much more heat is transferred. The heat transferred is

$$\dot{Q} = \dot{m} C_P \Delta T, \tag{10}$$

where ΔT is the temperature change of the gas on entering the heat exchanger. This convective heat transfer tracks the gas temperature near the heat exchanger.

The sequences shown in Fig. 3 are for a pulse tube cooler with an inertance tube. The sequence is reversed for an orifice pulse tube cooler. The heat transfer for both configurations is illustrated in Fig. 4. The illustrations combine both conduction and convection. The illustrated amplitudes of the two terms are arbitrary. The phase shifts between the mass flow and pressure used in Fig. 4 are given in Table 1 and were chosen to be near optimal for each configuration.[6]

The heat transfer is not a simple function expressible in the form assumed in the small amplitude approximation, Eq. (5). The conduction term can be approximated by Eq. (5). The convection term only lasts half a cycle. It is of the form:

for $\dot{m}$ into heat exchanger $$\dot{Q} = \dot{q}_0 + \dot{q}_1 \sin\left(2\omega t + \varphi\right) \tag{11a}$$

and for $\dot{m}$ out of heat exchanger $$\dot{Q} = 0, \tag{11b}$$

where $\dot{q}_0$, $\dot{q}_1$, and φ are interdependent constants with the constraint that $\dot{Q}$ is continuous. The convection term represents the conversion between the enthalpy flow in the pulse tube and heat exchanged to the heat exchanger. Thus, there is a transition region near the heat exchanger where there is a transition from pure enthalpy flow of the form of Eq. (5) to the form of Eq. (11). This transition region is of the order of the distance a gas element penetrates the pulse tube during a cycle. Evidence of this transition may have been seen in temperature measurements in the volume between a compressor and a regenerator.[7]

Because of the asymmetry in Eq. (11), there is a net heat flow between the gas and the heat exchanger and the mean gas temperature near the end of the pulse tube differs from the heat exchanger's temperature. The contribution to the heat transfer from the convection term is $\dot{q}_0/2$. Conduction decreases the heat transfer and temperature offset at both heat exchangers. The mean temperature offsets increases the heat conducted in the central region of the pulse tube.

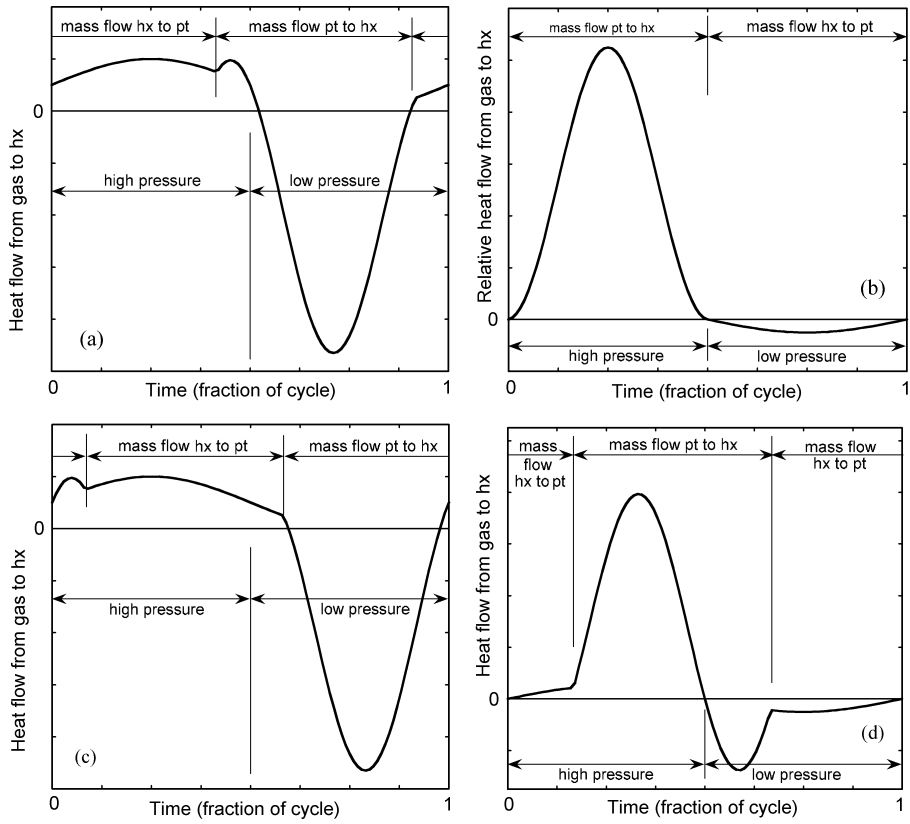

Figure 4. Qualitative representations of the heat flow from the working fluid (gas) to the heat exchangers for an orifice pulse tube cooler (a, b) and an inertance tube pulse tube cooler (c, d). The transition at the cold heat exchanger (a, c) and the hot heat exchanger (b, d) are shown. The regions of P P_0 and $P < P_0$ are indicated, as are the regions where the mass flows from the pulse tube (pt) to the heat exchanger (hx) and vice versa.

REGENERATOR TEMPERATURE PROFILE

Ideal case

In the regenerator the temperature oscillations and hence the enthalpy flow are suppressed by the intimate contact between the regenerator and the gas flow. The system exergy is carried by the entropy flow. At the ends, there are heat exchangers: the aftercooler and the cold heat exchanger. The entropy enters at the cold heat exchanger, flows through the regenerator, and exits at the aftercooler. This description defines the gross cooling power at the cold heat exchanger and the gross heat rejected at the hot heat exchanger:

$$\dot{Q}_c/T_c = \dot{Q}_a/T_a. \tag{12}$$

Again, this approach of considering only external heat flows does not define the temperature gradient. In the ideal case, there is no enthalpy flow; thus, Eq. (1) for a differential control volume of the regenerator reduces to $\nabla \dot{Q} = 0$ along the regenerator. The heat flows in a complex manor through both the regenerator material and the gas. This results in

$$\nabla \langle T \rangle \propto k_{eff}^{-1}. \tag{13}$$

where k_{eff} is a weighted average of the conductivity of the regenerator material, the gas, and the interfaces between them.[8] This average is a function of temperature and geometry and, thus, varies along the length of the regenerator.

Non-ideal effects

In real regenerators, there is a small enthalpy flow caused by imperfect heat exchange between the gas and regenerator material. Thus, one can expect to see end effects similar to those described above for the pulse tube. Since the enthalpy flow is small, the end effect is expected to small also. Two-dimensional CFD modeling has shown this effect to be insignificant.[5]

Several mechanisms result in an entropy gradient in the regenerator. These include thermal conduction, the pressure gradient, and the enthalpy flow.[2] None of these affects the temperature profile.

Real gas effects

The effects at low temperatures of real gas properties on the performance of pulse tube cryocoolers have been discussed previously and are summarized here.[3, 9] The real gas properties result in an enthalpy flow in the regenerator:

$$\dot{H} = \eta \dot{m}_1 P_1 \cos(\theta), \tag{14}$$

where $\eta = -\mu C_P$ and θ is the phase between the mass flow and pressure. At temperatures very much greater than the critical temperature, $\eta = 0$. For temperatures near the critical point, this enthalpy flow is considerable.

By combining Eqs. (8) and (12) one can see the effect on the temperature gradient. Fig. 5 shows the real gas effect on the regenerator's temperature profile near the critical point. There are two different solutions to the temperature gradient:

a) The temperature profile is flat at the cold heat exchanger. This occurs when the real gas effects are large compared to the nominal thermal conduction. The changing enthalpy flow results in the refrigeration occurring inside the regenerator rather than at the cold heat exchanger.[3]
b) The temperature continues to drop through out the regenerator. Here the real gas effects are smaller, the heat flow is always positive, and there is some refrigeration available at the cold heat exchanger.

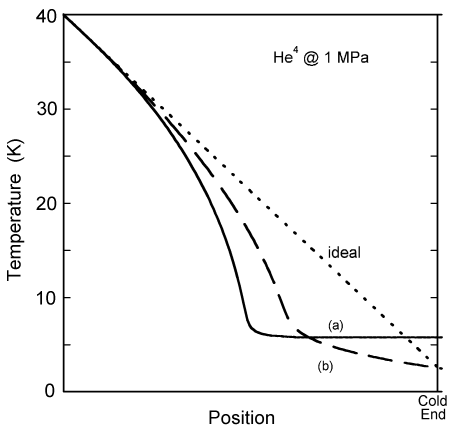

Figure 5. Regenerator temperature profiles when (a) enthalpy flow is greater and (b) thermal conductivity is greater.

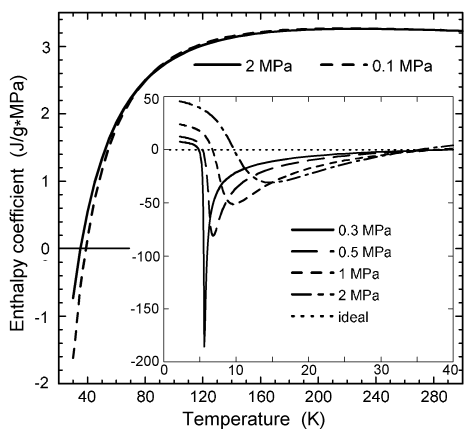

Figure 6. Enthalpy coefficient, η, for helium flow in a regenerator. For ideal case, η = 0. The maximum value is at ≈ 220 K. The low temperature behavior is inset.

What is often not realized is that the real gas effects extend to quite high temperatures. Fig. 6 shows the enthalpy coefficient, η, as a function of temperature.[10] Compared to its behavior near the critical point (inset Fig. 6), η is considerable smaller and pressure independent above 40 K.

However, the enthalpy flow for T > 40 K is sufficient for its gradient to significantly affect the temperature gradient. All four quantities on the right hand side of Eq. (12) vary with position. The pressure amplitude, P_1, decreases due to viscosity by a about 50% from the warm to cold ends of the regenerator. The mass flow, $\dot{m}_1$, also decreases from the warm to cold ends. The decrease is smaller than for the pressure and is due to void volume effects. Both effects also cause the phase, θ, to change. The gradient in η is the result of the temperature gradient. Combining these four gradients results in an enthalpy flow gradient and, thus, a heat flow gradient:

$$\nabla \dot{Q} = - \nabla \dot{H} \neq 0. \tag{15}$$

As a result, the temperature gradient will be less at the hot end and greater at the cold end than for the ideal case.

SUMMARY

The role thermoconductivity plays in pulse tube cryocoolers has been explored. It is critical to the operation of such coolers. It determines not only the temperature profiles in the regenerator and pulse tube but also interacts with real gas effects and influences the behavior at the transitions between components. The principal results discussed here are

1) For the pulse tube to heat exchanger transitions:
    a. The small amplitude model breaks down at these transitions as a result of the asymmetry in the heat transfer for different flow directions.
    b. The asymmetrical heat transfer results in higher than expected temperature overshoots/undershoots in the gas near the pulse tube ends. This also causes increased entropy generation due to transferring heat over increased temperature differences
2) For the pulse tube:
    a. The temperature gradient in the center region of the pulse tube is determined by thermal conductivity.
    b. The temperature gradient is greater than expected due to the temperature over/under-shoots at the ends of the pulse tube.
3) For the regenerator:
    a. At temperature near the critical point, real gas effects dominate and can limit the zero load temperature.
    b. For temperatures above 40 K, the temperature profile is dominated by thermal conductivity.
    c. Real gas effects result in an enthalpy flow in the regenerator to temperatures well above 300 K. This produces a correction to the temperature profile.

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