Theoretical Analysis and Experimental Validation of a Pulse Tube with a Cold Reservoir

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ABSTRACT
In order to improve the cooling performance of pulse tube cryocoolers (PTC), a novel configuration is proposed where a reservoir is added through an orifice valve to the cold end of pulse tube. The additional resistant element to the cold end helps to directly introduce the mass flow rate parallel to the pressure wave, which is beneficial to the cooling capacity. Thermodynamic relations between the cooling capacity, the coefficient of performance and various parameters were deduced for the novel configuration in orifice type and double-inlet type PTC with cold reservoir. Theoretical analysis shows that the cooling performance of the PTC can be improved significantly with the addition of a cold reservoir. Experiments with a home-made single stage Gifford-McMahon (GM) type PTC with a cold reservoir were also implemented for validation. Preliminary experimental results reveal that the cooling capacity of PTC was improved with minor opening of cold-end orifice valve. There exists an optimal opening of the cold-end orifice valve to improve the cooling performance.

INTRODUCTION
The pulse tube cryocooler (PTC) is used widely because of its low vibration, long mean-time between maintenance (MTBM) and low electromagnetic interference (EMI) characteristics due to the absence of moving parts at cryogenic temperatures. According to the harmonic analysis of pulse tubes by Radebaugh, the cooling power of a PTC depends closely on the phase difference between the pressure wave and the mass flow rate at the cold end of pulse tube. The maximum cooling capacity can be achieved when the pressure and mass flow rate are in phase, while no cooling power will be produced if these two are orthogonal to each other. It is clear that to improve the cooling capacity of PTC, the parallel component of the flow rate to the pressure wave at the cold end of pulse tube should be increased, while the orthogonal component should be compressed.

Since the invention of the pulse tube cryocooler by Gifford and Longsworth in 1963, many improvements have been made to adjust the phase. In 1986, Mikulin et al. added an orifice and a reservoir at the hot end of pulse tube, this introduces a parallel and an orthotropic flow component, respectively, to the pressure oscillation at the cold end of pulse tube. The parallel volumetric flow rate to the pressure wave added the PV power flow which equals to the cooling power for an ideal thermodynamic process, and the cooling performance was greatly improved compared to the basic
type. Whereas, the accompanied orthogonal component consumed the compressing work instead of contributing to the cooling power in a cycle, and was disadvantageous to the coefficient of performance (COP). Another major improvement was the addition by Zhu et al. of a bypass valve connecting the hot end of the regenerator and the hot end of the pulse tube, named double-inlet pulse tube refrigerators. Since the magnitude of the parallel component of flow rate through the bypass valve is restricted by the phase difference between the hot and cold end of the regenerator, thus the phase adjusting function of the bypass valve is not much. Similarly, the configuration of the multi-pass type PTC helps to decrease the total pressure drop through the regenerator, and to increase the pressure amplitude inside the pulse tube, which leads to better cooling performance. However, it contributes little to the phase adjusting. Chen et al. also proposed a phase shifter termed a double-orifice, with which the the lowest temperature of 3.1 K at the cold end of the second stage was achieved.

The above-mentioned phase adjusting methods for the cold end of PTC are indirect, i.e. to optimize the phase indirectly through the phase adjusting parts at the hot end to obtain the ideal phase difference at the cold end of pulse tube. This study proposed a new configuration of PTC, which adds a reservoir through a valve to the cold end of pulse tube, termed cold end reservoir type PTC. The combination of an orifice and a reservoir can bring in-phase component of the mass flow rate to the pressure wave, so the phase difference at the cold end can be adjusted directly and the cooling performance is promising to be improved. The theoretical analysis was applied by the linear model analysis for both the single orifice PTC and the double-inlet PTC with a cold reservoir. The dependence of cooling performance on the cold valve opening for both PTCs were presented. Preliminary experimental studies with a single stage GM type PTC also validated the performance of the new configuration.

**THEORETICAL MODEL**

**Single Orifice PTC**

The simplified model of analysis for the single orifice PTC is presented in Figure 1. A cold reservoir through a needle valve is connected to the conventional orifice PTC. There will be a certain in-phase component of mass flow rate to the pressure wave introduced by the cold-end orifice and reservoir which are predicted to improve the cooling performance. The working flow at the hot end of regenerator is at temperature $T_h$, and that at the cold end is at $T_c$. The basic assumptions made in this theoretical analysis are:

1. Ideal gas and one-dimensional flow;
2. The temporal pressure in pulse tube, the hot end of regenerator, and the two reservoirs is sinusoidal;
3. The pressure amplitude is far smaller than the time-averaged pressure;
4. The volumetric flow rate through regenerator, the bypass valve and the two orifice valve is proportional to the pressure difference across them;
5. The void volume in regenerator is neglected;
6. The hot and cold reservoir and pulse tube are adiabatic.

![Figure 1. simplified model used for analysis of single orifice PTC with a cold reservoir](image-url)
From the assumption (2), the pressure $p_p$ in pulse tube as well as $p_h$ in the hot reservoir and $p_c$ in the cold reservoir is:

$$p_p(t) = \Delta p_p \sin(\omega t) + p_0$$

$$p_h(t) = \Delta p_h \sin(\omega t + \delta_h) + p_0$$

$$p_c(t) = \Delta p_c \sin(\omega t + \delta_c) + p_0$$

From the assumption (4), the mass flow $\dot{m}_h$ and $\dot{m}_c$ through the hot and cold-end orifice valve can be displayed as:

$$\dot{m}_h = \rho_h C_h [p_p(t) - p_h(t)]$$

$$\dot{m}_c = \rho_c C_c [p_p(t) - p_c(t)]$$

where $C_h$ and $C_c$ are the corresponding flow conductance across valves.

For the hot reservoir, it can be assumed that the flow in the reservoir is polytropic, so the variation of the pressure in the hot reservoir can be given as:

$$\pi_h \cos(\omega t + \delta_h) = (F \cdot \Omega_V h)^{-1} OC_h [\sin(\omega t) - \pi_h \sin(\omega t + \delta_h)]$$

where $\pi_h \equiv \Delta p_h / \Delta p_p$ and $\Omega_V h \equiv V_h / V_p$ are the amplitude and volumetric ratio of the hot reservoir to the pulse tube. $OC_h \equiv C_h / C_r$ is the flow conductance ratio of the hot-end orifice valve to the regenerator.

The dimensionless frequency is defined as $F \equiv \omega V_p / k \rho_0 C_r$.

Equating the coefficient of terms $\cos(\omega t)$ and $\sin(\omega t)$ on the two sides of Eq. (7) yields:

$$\tan \delta_h = - \frac{\pi_h}{\pi_c} = \frac{OC_c}{OC_h}$$

In the same manner, for the cold reservoir, the flow can also be assumed as polytropic and the corresponding relations can be given as:

$$\tan \delta_c = - \frac{\pi_h}{\pi_c} = \frac{OC_c}{OC_h}$$

where $\pi_c \equiv \Delta p_c / \Delta p_p$, $\Omega_V c \equiv V_c / V_p$ and $OC_c \equiv C_c / C_r$.

Substituting Eq. (1)-(3) into Eq. (4) and (5), the mass flows through the hot and cold-end orifice valves connecting to the two reservoirs can be displayed as:

$$\dot{m}_h = \rho_h C_h [p_p(t) - p_h(t)] = \rho_h C_h \Delta p_p [\sin(\omega t) - \Omega_V h \sin(\omega t + \delta_h)]$$

$$\dot{m}_c = \rho_c C_c [p_p(t) - p_c(t)] = \rho_c C_c \Delta p_p [\sin(\omega t) - \Omega_V c \sin(\omega t + \delta_c)]$$

The volumetric flow rate $V_{pt}$ at the cold end of pulse tube can be expressed as the summation of the flow rates in pulse tube and two reservoirs:

$$V_{pt} = \frac{V_p}{\kappa p_0} \frac{dp_p(t)}{dt} + V_h + V_c = \frac{V_p}{\kappa p_0} \omega \Delta p_p \cos(\omega t) + \frac{\dot{m}_h}{\rho_h} + \frac{\dot{m}_c}{\rho_c}$$

Substituting Eq. (12) and (13) into Eq. (14), then the volumetric flow rate at the cold end of pulse tube is:

$$V_{pt} = \Delta p_h C_h \{F \cos(\omega t) + OC_h \left[ (1 - \pi_h^2) \sin(\omega t) - \pi_h \sin \delta_h \cos(\omega t) \right]$$

$$+ OC_c \left[ (1 - \pi_c^2) \sin(\omega t) - \pi_c \sin \delta_c \cos(\omega t) \right]$$

$$+ \frac{\dot{m}_h}{\rho_h} + \frac{\dot{m}_c}{\rho_c} \}$$
The first term on the right hand side of the above equation represents the flow rate variance due to the movement of the gas piston in pulse tube, while the next two terms are the flow rate through the hot-end and the cold-end orifice valve. Obviously, the first term comprises only the cosine component which is always orthogonal to the pressure wave, and does not contribute to the cooling power for PTC; while the other two terms have the sine and cosine components, and the sinusoidal one is contributes the most to the cooling power. For the ideal thermodynamic processes, according to de Boer\textsuperscript{7}, the cooling power can be expressed as:

\[ \dot{Q}_c = \langle h \rangle = \langle p(t) \dot{V}_{pt} \rangle \]  

Carrying out the integral and using Eq. (1) and (14), the dimensionless cooling power can be given as:

\[ \dot{Q}_0 = \frac{\langle h \rangle}{c_p p_0^2} = OC_h (1 - \pi_h^2) + OC_c (1 - \pi_c^2) \]  

Considering the mass flow gets through the regenerator to the cold end of pulse tube, and assuming that the temporal pressure at the hot end of regenerator is \( p(t) \), the volumetric flow rate across the regenerator can be calculated as:

\[ \dot{V}_{pt} = c_c [p(t) - p_h(t)] \]  

Using Eq. (1) and (15), then the pressure at the hot end of regenerator is given as:

\[ p(t) = \Delta p_p \left[ \frac{f \cos(\omega t)}{c_r} + OC_h (1 - \pi_h^2) \sin(\omega t) - \pi_h \sin \delta_h \cos(\omega t) \right] + OC_c (1 - \pi_c^2) \sin(\omega t) - \pi_c \sin \delta_c \cos(\omega t)] + \Delta p_p \sin(\omega t) + p_0 \]  

From the assumption (5), the volumetric flow rate through the regenerator is calculated as:

\[ \dot{V}_r = \frac{T_h}{T_c} \dot{V}_{pt} \]  

combining Eq. (19) and (20) above, the compressing power is obtained as:

\[ \dot{p}_d = \langle p(t) \dot{V}_r \rangle = \frac{c_r}{T_c} \cdot \frac{T_h}{T_c} \cdot \Delta p_p^2 \left[ \frac{OC_h (1 - \pi_h^2)}{1 + OC_h (1 - \pi_h^2)} \cdot \left(1 + OC_c (1 - \pi_c^2)\right) \right] + \Delta p_p \left( F - \pi_h \sin \delta_h - OC_c c_c \sin \delta_c \right)^2 \]  

So the coefficient of performance of single orifice PTC with a cold reservoir can be expressed as:

\[ \text{COP} = \frac{\langle h \rangle}{\dot{p}_d} = \frac{T_c}{T_h} \cdot \frac{OC_h (1 - \pi_h^2) + OC_c (1 - \pi_c^2)}{OC_h (1 - \pi_h^2) + OC_c (1 - \pi_c^2) \cdot \left(1 + OC_h (1 - \pi_h^2) + OC_c (1 - \pi_c^2)\right) + \Delta p_p \left( F - \pi_h \sin \delta_h - OC_c c_c \sin \delta_c \right)^2} \]  

For the ideal orifice type PTC without the cold reservoir, \( OC_c = 0 \) and \( \pi_c = 0 \). Neglecting the pressure fluctuation in the hot reservoir, \( \pi_c \to 0 \), then Eq. (22) can be rewritten as:

\[ \text{COP} = \frac{T_c}{T_h} \cdot \frac{OC_h}{OC_h (1 + OC_h) + F^2} \]  

which is the same to the theoretical calculations obtained by de Boer\textsuperscript{8}.

**Double-inlet PTC**

For the case of double-inlet type PTC, a bypass tube through a bypass valve connects the hot ends of the pulse tube and the regenerator, as shown in Figure 2.
In theoretical analysis, Eq. (1)-(13) are still applied, while from the assumption (4), the mass flow rate across the bypass valve is expressed as:

$$ \dot{m}_b = \rho_b C_b [p(t) - p_p(t)] $$ (24)

and $C_b$ is the flow conductance of the bypass valve. According to the mass flow conservation, the volumetric flow rate at the cold end of the pulse tube in double-inlet PTC with a cold reservoir is the summation of those through the hot-end orifice valve, bypass valve $V_p$, cold-end orifice valve and the movement of the gas piston, leads to:

$$ \dot{V}_p = \frac{V_p}{\kappa_p} \frac{d p_p(t)}{dt} + \dot{V}_h + \dot{V}_c - \dot{m}_b = \frac{V_p}{\kappa_p} \omega \Delta p_p \cos(\omega t) + \dot{m}_b + \dot{m}_c - \dot{m}_b $$ (25)

Substituting Eq. (12), (13) and (24) into Eq. (25), then the temporal pressure at the hot end of the regenerator can be calculated as:

$$ p(t) = \frac{\Delta p_p [F \cos(\omega t) + O_{Ch} (1 - \pi_h^2)] \sin(\omega t) - \pi_h \sin \delta_h \cos(\omega t)] + O_{Cc} [(1 - \pi_c^2) \sin(\omega t) - \pi_c \sin \delta_c \cos(\omega t)]}{1 + DC} + p_0 $$ (26)

and $DC \equiv \frac{C_b}{C_r}$ is the flow conductance ratio of the bypass valve to the regenerator. Combining the Eq. (24-26), the volumetric flow rate at the cold end of the pulse tube can be obtained:

$$ \dot{Q}_0' = \frac{\pi_h}{C_d p_0^2} = \frac{O_{Ch} (1 - \pi_h^2) + O_{Cc} (1 - \pi_c^2)}{1 + DC} $$ (27)

Like the single orifice type PTC, the dimensionless cooling power can be expressed from the Eq. (27):

$$ \dot{Q}_0' = \frac{\pi_h}{C_d p_0^2} $$ (28)

According to the mass conservation and the assumption (5), the volumetric flow rate at the hot end of the regenerator can be calculated as:

$$ \dot{V}_r = \frac{\pi_h}{\pi_c} \dot{V}_p + \dot{V}_b = \frac{\pi_h}{\pi_c} \frac{\Delta p_b C_r [F \cos(\omega t) + O_{Ch} (1 - \pi_h^2)] \sin(\omega t) - \pi_h \sin \delta_h \cos(\omega t)] + O_{Cc} [(1 - \pi_c^2) \sin(\omega t) - \pi_c \sin \delta_c \cos(\omega t)]}{1 + DC} $$ (29)

Then the compressing power for the double-inlet type PTC with a cold reservoir can be given as:

$$ p'_c = \frac{p(t) \dot{V}_r'}{\frac{C_d \Delta p_h}{2(1 + DC) \pi_c} + \frac{\pi_h}{\pi_c} \cdot (\frac{\pi_h}{\pi_c} + DC) \cdot (O_{Ch} (1 - \pi_h^2) + O_{Cc} (1 - \pi_c^2))} $$ (30)

The coefficient of performance for the PTC can be deduced by combining the Eq. (28) and (30):

$$ \dot{Q}'_c = \frac{\frac{O_{Ch} (1 + DC)}{1 + DC + O_{Ch} + F \pi_h \sin \delta_h - O_{Cc} \pi_c \sin \delta_c}^2}{\frac{O_{Ch} (1 + DC)}{(1 + DC + O_{Ch} + F \pi_h \sin \delta_h - O_{Cc} \pi_c \sin \delta_c)^2}} $$ (31)

For the ideal double-inlet type PTC without the cold reservoir, in which the pressure fluctuation at the hot reservoir, the Eq. (31) can be rewritten as:

$$ \dot{Q}'_c = \frac{O_{Ch} (1 + DC)}{1 + DC + O_{Ch} + F \pi_h \sin \delta_h - O_{Cc} \pi_c \sin \delta_c}^2 $$ (32)

which is the same to the theoretical calculations obtained by de Waele.9

**COMPUTATIONAL RESULTS AND ANALYSIS**

The dimensionless cooling power is plotted in Figure 3 (a) as a function of the volumetric ratio $OV_c$ for the single orifice PTC. Compared to the basic condition of a conventional orifice PTC, the cooling power of the PTC with a cold reservoir is much larger than the basic one, and it rises with the increase of $OV_c$. However, as the $OV_c$ gets larger, the slope of $\dot{Q}_0$ decreases and the cooling
power becomes constant for large $OV_C$. The dependence of COP on $OC$ is also displayed in Figure 3 (b). Since both the parallel component and the orthotropic one of the flow consume the input power, the COP depends on the combination of both two parts. For certain flow conductance ratio $OC_C$, when the $OV_C$ is small, the orthotropic flow overwhelms the parallel one, and the COP gets lower than that of the conventional PTC; however, as the $OV_C$ rises, the parallel flow becomes dominant and the COP increases and exceeds the basic case of a conventional PTC.

Like the orifice PTC, the dimensionless cooling power and COP for the double-inlet PTC are also plotted as the functions of $OV_C$ in Figure 4 (a) and Figure 4 (b). For the given condition as $F=0.5$, $OC_C=0.2$, $OV_h=10$, $DC=0.1$ and large $OV_C$, the double-inlet PTC with a cold reservoir also shows superiority over the conventional one in the cooling power and COP. Comparing Figure 3 (b) and Figure 4 (b), it reveals that for certain $OC_C$, the COP of double-inlet PTC with the cold reservoir is larger than that of the orifice type with a cold reservoir.

**EXPERIMENTAL VALIDATION**

To further validate the superiority of the new configuration, preliminary experiments have been implemented with a home-made PTC. Figure 5 shows the schematic of a single GM type PTC with a cold reservoir connected through an orifice valve to the cold end of pulse tube.

A photograph of the corresponding new PTC is displayed in Figure 6 (a). The pulse tube and regenerator are fabricated from stainless steel tubes with outer diameter, wall thickness and length

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**Figure 3.** (a) dimensionless cooling power dependence on volumetric ratio of cold reservoir to the pulse tube for a single orifice PTC with a cold reservoir; (b) the coefficiency of performance dependence on volumetric ratio of cold reservoir to the pulse tube for a single orifice PTC with a cold reservoir.

**Figure 4.** (a) dimensionless cooling power dependence on volumetric ratio of cold reservoir to the pulse tube for a single orifice PTC with a cold reservoir; (b) the coefficiency of performance dependence on volumetric ratio of cold reservoir to the pulse tube for a single orifice PTC with a cold reservoir.
as follows: pulse tube: $\phi 16 \text{mm} \times 0.3 \text{mm} \times 250 \text{mm}$; regenerator: $\phi 20 \text{mm} \times 0.3 \text{mm} \times 210 \text{mm}$. The regenerator was alternately packed with the 247 meshes of stainless steel and phosphor-bronze screens. In this study, a common reservoir of 0.5 L was connected through an orifice valve to the hot end of pulse tube, while another reservoir of 0.45 L, termed cold reservoir, was connected to the U shape tube between the cold ends of regenerator and pulse tube through an orifice valve. A SS-ORS3MM needle valve produced by Swagelok was used as the cold-end orifice valve. The flow conductance ratio can be adjusted by the opening of the cold-end orifice valve.

The cryocooler operates in double-inlet mode with the anti-parallel arrangement to adjust the DC flow. Water-cooled helium compressor of SUMITOMO HEAVY INDUSTRIES with rated input power of about 4 kW was used to generate the pressure oscillation through a rotary valve. The refrigeration temperature of the cold head was measured by calibrated Rh-Fe resistance thermometer with an accuracy of 0.1 K, while calibrated Pt100 resistance thermometers were installed on the outer wall of pulse tube, regenerator and cold reservoir to monitor the temperature distribution. The cooling power was determined by measuring the electrical power input to a heater tightly attached to the cold-end of the pulse tube with the accuracy of 1.0 mW. The integrated apparatus is displayed in Figure 6 (b).

The system was initially charged to 1.25 MPa with pure helium and operated with frequency of 2 Hz. The only distinct aspect for the PTC with a cold reservoir is the cold-end orifice valve.

**Figure 5.** Schematic of the single stage GM type PTC with a cold reservoir

**Figure 6.** Photographs of (a) the single stage GM type PTC with a cold reservoir and (b) the experimental apparatus
Besides, a by-pass valve was connected between the inlet and outlet of the helium compressor, and the pressure ratios in the hot end of regenerator were maintained on the same level during the tests.

Various cooling capacities of the PTC with different openings of cold-end orifice valve have been measured to determine the effects of the new configuration to cooling performance. As shown in Figure 7 (a), the case that is considered as the basic condition. The experimental results show the following points:

1. When $O_2$ is minor, cooling capacity of novel PTC with the cold reservoir is improved in comparison with that of the basic one, just as the theoretical analysis above. However, as $O_2$ gets larger, the no-load temperature and cooling capacity deteriorate. Unlike the thermodynamic calculations, the opening of cold-end orifice valve has significant effect on the cooling capacity. With the large $O_2$, more mass flow is required during the adiabatic expansion to improve the cooling performance, whereas, the diameter of regenerator is inadequate in the experiments that regenerator losses rise, which decreases the cooling capacity. So the improvement of cooling capacity happens during minor $O_2$.

2. Figure 7 (b) displays the cold head temperature with different cooling capacities depend on $O_2$. As the theoretical analysis, there exists an optimal $OC_c$, which is embodied by $O_2$ in experiments, for the cooling performance. When $O_2=1/40$ turn, the cold head temperature is lowest with $Q_c=1.1\text{W}$, $2.5\text{W}$, $4.4\text{W}$, while $T_c$ increases with the rise of $O_2$.

3. Introducing a cold reservoir to the cold head of PTC also increases the cooling load, and the heat from adiabatic compression should be taken away through regenerator and pulse tube. In the experiments, additional heat load significantly affects the cooling time of PTC. Cooling time of cold head of PTC with large $O_2$ is significantly extended than those with minor $O_2$. However, with minor $O_2$, the cooling down of regenerator and cold reservoir is much slower than those with large $O_2$, due to small mass flow rate joins the adiabatic expansion in the cold reservoir.

CONCLUSIONS

To increase the cooling performance of a PTC, a cold reservoir through an orifice valve is added directly to the cold end of regenerator. With the addition of a resistant element, through which mass flow is parallel to the pressure wave, refrigeration power per unit flow rate is increased. Meanwhile, the cold reservoir introduces more mass flow through regenerator.

The calculation results show that cooling capacity and COP of novel PTC can be significantly improved, while there exists an optimal $OC_c$. Experiments with a home-made single stage GM type PTC added with a cold reservoir at the cold end between regenerator and pulse tube also verify the

Figure 7. (a) comparison of cooling capacities with different opening of the cold-end orifice valve;(b) variation of cold head temperature of PTC with different opening of the cold-end orifice valve and cooling capacities
design. The experimental results show that cooling capacity of novel PTC is improved with minor opening of the cold-end orifice valve, which agrees with the theoretical prediction. However, with the larger $O_2$, the cooling performance deteriorates. With the cooling load $Q_c = 1.1W, 2.5W, 4.4W$, lowest cold head temperature can be reached with $O_2 = 1/40$ turn.

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