Simulation of Thermodynamic Cycles in Linear Type Pulse Tube Cryocoolers

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ABSTRACT

In this study, a simulation is developed to analyze gas parcels in a linear type pulse tube cryocooler (PTC). Thermodynamic processes of gas parcels in the regenerator, the heat exchanger and the pulse tube are considered in the simulation. The pressure in the pulse tube is assumed to be sinusoidal. The working fluid is assumed to be an ideal gas. Length-wise mixing and heat conduction are ignored in the simulation, and the thermodynamic processes in the pulse tube are considered to be adiabatic. By using the Lagrange method, the simulation follows the thermodynamic processes of each gas parcel and computes the thermodynamic parameters of the working fluid throughout the processes. The refrigeration capacity is also calculated in the simulation. The results of the simulation are helpful for designing PTCs and studying the physical mechanism of the working fluid in the PTC.

INTRODUCTION

PTC has been widely used in infrared detection over the past 10 years. As the infrared detection technology developed, more stringent requirements have been proposed for PTC. The study of alternating flow in the PTC is the key to improving the performance of PTC. Experimental research is one of the methods used to study the nature of alternating flow in PTCs, and many improvements have been made through this type of research. However, there are still some disadvantages. The first one is that some parameters are difficult to measure by experimentation, e.g., the velocity in the regenerator. The second one is that experimental research takes time and is expensive. In recent years, as computer technology has developed, higher performance computers allow us to simulate PTCs with more complex details. It makes simulation research an important research method in the PTC technology field.

Many research studies have been conducted with the simulation method. Most of them are done using software tools like FLUENT and CFX. An advantage of commercial software is its high reliability, but in commercial software the calculation process is invisible and uncontrollable. In this study, a program based on the differential element method was developed to simulate thermodynamic cycle of fluid in a 1-D PTC. The inputs and the calculating method are all controllable. The results give the position, temperature and pressure of each gas parcels at any time step. The nature of the alternating flow in the PTC is analyzed with these results. The changes in the refrigeration power with a change in the cold heat exchanger temperature are presented.

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Nomenclature

A	cross sectional area of gas flow (m^2)
A_f	heat transfer area of fin (m^2)
A_t	total heat transfer area of heat exchanger (m^2)
D	diameter (m)
L	length (m)
L_c	equivalent height of fin (m)
P	pressure (Pa)
Q_c	cooling power (W)
Q_t	heat transfer between gas and solid (W)
R	gas constant
Re	Reynolds number
T	temperature (K)
U	internal power (w)
V	volume (m^3)
W	power (w)
b	thickness of gas parcel (m)
c	conductance coefficient of orifice (kg Pa^{-1} s^{-1})
c_v	constant volume specific heat (J kg^{-1} K^{-1})
c_p	constant pressure specific heat (J kg^{-1} K^{-1})
d_h	hydraulic diameter (m)
f	resistance coefficient
h	heat transfer coefficient (W m^2 K)
k	adiabatic exponent
k_t	thermal conductivity (W m^{-1} K^{-1})
m	mass (kg)
$\dot{m}$	mass flow rate (kg s^{-1})
n	number of fins in heat exchanger
t	time (s)
u	velocity (m s^{-1})
x	longitudinal coordinate (m)

Greek letters

α	angel between mass flow and pressure
δ	thickness of fin (m)
δm	mass of gas parcel (kg)
δt	time step (s)
η_f	heat exchange efficiency of a single fin
ω	angular frequency

Subscripts

A	ambient
c	cold end of pulse tube
ex	heat exchanger
h	hot end of pulse tube
i	time step number
j	parcel number
re	regenerator
w	wall
pt	pulse tube

Abbreviations

AC	alter cooler
B	buffer
CHX	cold heat exchanger
HHX	hot heat exchanger
Ori	orifice
Reg	regenerator

PHYSICS MODEL

A one-dimensional model of a linear type pulse tube cryocooler is developed. The model is shown in Figure 1. It includes after cooler (AC), regenerator (Reg), cold heat exchanger (CHX), pulse tube (PT), hot heat exchanger (HHX), orifice (Ori) and buffer (B).

Pulse Tube Model

In the pulse tube model, the following assumptions are made:

(1) The working fluid is considered an ideal gas;
(2) Gas experiences an adiabatic process in the pulse tube;
(3) The pressure in the pulse tube is sinusoidal;
(4) The axial heat conduction of the gas is negligible.

When taking the pulse tube as the control-volume, the conservation equation can be described as Eq. (1):

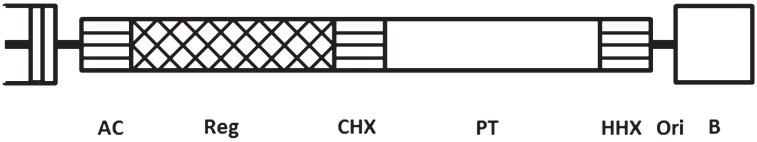

Figure 1. Schematic diagram of PTC model

$$\frac{c_v A_{pt}}{R}\frac{dp}{dt}+\frac{c_p}{R}\frac{\partial(p\dot{V})}{\partial x}=0 \tag{1}$$

The state equation of ideal gas can be expressed as follows:

$$p\dot{V}=\dot{m}RT \tag{2}$$

Introducing Eq. (2) to Eq. (1), and integrating with respect to x, Eq. (3) is obtained:

$$\dot{m}_c=\frac{T_h}{T_c}\dot{m}_h+\frac{V_{pt}}{kRT_c}(\frac{dP}{dt}) \tag{3}$$

In Eq. (3), T_c is the average temperature of gas at the cold end of the pulse tube, T_h is the average temperature of gas at the hot end of pulse tube, $\dot{m}_c$ is the mass flow rate at the cold end of the pulse tube, $\dot{m}_h$ is the mass flow rate at the hot end of the pulse tube.

Pressure in the pulse tube can be expressed as follows:

$$P_{pt}(t)=P_{av}+P_A\cdot\sin(\omega\cdot t) \tag{4}$$

Assuming the pressure in the gas reservoir is equal to the average pressure in the pulse tube P_{av}, $\dot{m}_h$ can be expressed as:

$$\dot{m}_h=c\cdot(P_{pt}-P_{av}) \tag{5}$$

Combining Eq. (3) to Eq. (5), the following equation can be deduced:

$$\dot{m}_c=[(\frac{cT_hP_A}{T_c})^2+(\frac{\omega V_{pt}P_A}{kRT_c})^2]^{\frac{1}{2}}\cdot\sin[\omega\cdot t+\arctan(\frac{\omega V_{pt}}{ckRT_h})] \tag{6}$$

It is obvious that $\dot{m}_c$ is also sinusoidal. In the first half of the cycle, gas goes into the pulse tube, and in the second half of the cycle, gas goes out of the pulse tube.

The thermodynamic non-symmetry effect theory, proposed by J. Liang [1], is used to analyze the thermodynamic processes here. The theory is based on the Lagrange method. According to the theory, the first half cycle is made up of n equal time steps. The gas goes into the pulse tube at time step j and is marked as gas parcel j, the mass of gas parcel j is:

$$\delta m_j=\int_{t_{j-1}}^{t_j}\dot{m}_c\,dt \tag{7}$$

The total number of gas parcels that enter PT is n. Since all the gas parcels experience adiabatic processes in the pulse tube, the temperature of gas parcel j at time step i is as follows:

$$T_j(t_i)=T_c(\frac{P(t_i)}{P(t_{j-1})})^{\frac{k-1}{k}} \tag{8}$$

If the position of the cold end of the pulse tube is defined as the zero point of x, the position of the gas parcel j at time step i can be expressed as:

$$\mathrm{x}_j(t_i)=\frac{1}{A_{pt}}\sum_{j=J}^{i}V_j(t_i) \tag{9}$$

In the second half of the cycle, gas parcels leave the cold end of pulse tube, the time when gas parcel j leaves the pulse tube can be calculated by the following equation:

$$\int_{t_n}^{t_i}\dot{m}_c\,dt=\sum_{j=2n+1-i}^{n}\Delta m_j \tag{10}$$

Based on Eq. (8) to Eq. (10), the temperature, the pressure and the position of gas parcels at any time step in PT can be obtained.

HEAT EXCHANGER MODEL

The heat exchanger is considered as finned tube exchanger. The total heat transfer coefficient can be calculated by the following equation:

$$h_t=h\cdot A_f\cdot[1-(\frac{N\cdot A_f}{A_t})\cdot(1-\eta_f)] \tag{11}$$

where A_f is the surface area of a single fin, and A_t is the total surface area of the heat exchanger. η_f is the heat exchange efficiency of fins which can be expressed as:

$$\eta_f = \tanh[(\frac{2h}{k_t\delta})^{1/2} \cdot L_c] \cdot [(\frac{2h}{k_t\delta})^{1/2} \cdot L_c]^{-1} \tag{12}$$

For gas parcel j in the heat exchanger, if its temperature at t_i is known, the heat absorbed from the wall at t_i can be calculated as:

$$q_j(t_i) = \delta t_i \cdot h_t \cdot [T_w - T_h^j(t_i)] \tag{13}$$

The energy conservation equation of gas parcel j at time step i can be given by Eq. (14):

$$\Delta U_j = q_j(t_i) + W_j(t_i) \tag{14}$$

Where $W_j(t_i)$ can be expressed as:

$$W_j(t_i) = \frac{[P(t_i) + \mathrm{P}(t_{i+1})]}{2} \cdot [V_j(t_i) - V_j(t_{i+1})] \tag{15}$$

Assuming the pressure in the heat exchanger is sinusoidal:

$$P_h(t) = P_{av} + P_A \cdot \sin(\omega \cdot t) \tag{16}$$

The state equation of ideal gas is:

$$V_j(t_i) = \frac{m_j R T_j(t_i)}{P(t_i)} \tag{17}$$

$$V_j(t_{i+1}) = \frac{m_j R T_j(t_{i+1})}{P(t_{i+1})} \tag{18}$$

Combining Eq. (13) to Eq. (18), the temperature of gas parcel j at t_{i+1} can be calculated as:

$$T_j(t_{i+1}) = [q_j(t_i) + (\bar{P} \cdot \frac{m_j R}{P(t_i)} + c_p m_j) \cdot T_j(t_i)] \cdot (\bar{P} \cdot \frac{m_j R}{P(t_{i+1})} + c_p m_j)^{-1} \tag{19}$$

The position of gas parcel j at time step i+1 can also be calculated by the following equation:

$$x_j(\mathrm{t}_{i+1}) = -\sum_{n=1}^{i} b(j+1-n) \tag{20}$$

Where b is the thickness of gas parcel.

With Eq. (19) and Eq. (20), if the initial temperature and position of a gas parcel is known, the temperature and position at any time step in the heat exchanger can be calculated.

REGENERATOR MODEL

In the regenerator, the following assumptions are made:

(1) The heat capacity of the solid in regenerator is infinite;
(2) The temperature of the regenerator is piecewise linear;
(3) Viscosity resistance in the regenerator is considered.

Gas parcel j in the regenerator is shown in Figure 2; L is the distance from gas parcel to the CHX. Since the pressure in the heat exchanger and pulse tube is sinusoidal, the pressure of gas parcel j can be calculated by the following equation:

$$P_j^{re}(t_i) = P_{pt}(t_i) + \Delta P_j(t_i) \tag{21}$$

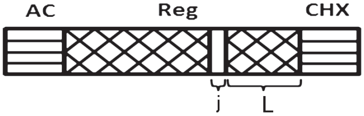

Figure 2. Gas parcel j in Reg

where $P_{pt}(t_i)$ is the pressure in pulse tube at time step i, $\Delta P_j(t_i)$ is the pressure drop from position $x_j^{re}(t_i)$ to the cold end of regenerator, which can be expressed as:

$$\Delta P_j(t_i) = f_j(t_i) \cdot \frac{\rho \cdot L_j(t_i) \cdot u_{av}(t_i)^2}{2D_h} \tag{22}$$

where $f_j(t_i)$ is the average resistance coefficient from position $x_j^{re}(t_i)$ to the cold end of the regenerator. $f_j(t_i)$ can be calculated by the following empirical equation which is proposed by W. Xi-long[2]:

$$f_j(t_i) = \frac{158.08}{Re_j(t_i)} + 0.04 \tag{23}$$

Combining Eq. (21) to Eq. (23), the pressure of gas parcels at any time step can be calculated. The temperature and positions of gas parcels in the regenerator can be achieved in the same way as in the heat exchanger.

MODEL INTEGRATION

By combining the PT, CHX and Reg models together, the thermodynamic processes of all gas parcels that enter the PT can be calculated.

There are some gas parcels that don't enter the PT, as shown in Figure 3. They only move between the AC and the CHX. The thermodynamic processes of these gas parcels can be calculated by using the parameters of gas parcel n, which is the last one to enter the PT, as the boundary condition.

SIMULATION RESULT

With the 1-D model, calculations have been performed to analyze thermodynamic cycles of gas parcels in a linear type pulse tube cryocooler. Geometrical parameters of the pulse tube cryocooler are shown in Table 1. Simulations are done under the following conditions: frequency is 50Hz, filling pressure is 1.5 MPa, temperature of CHX is 80K and the total number of gas parcels that enter PT is 150. The temperature gradient of the solid along the axial direction is shown in Figure 4.

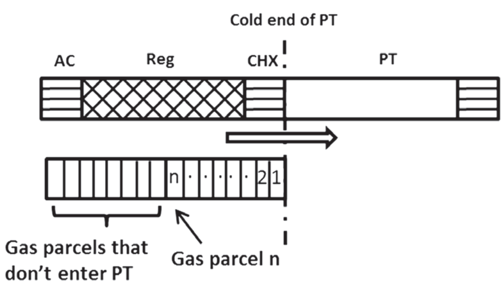

Figure 3. Schematic diagram of gas parcels in the PTC

Table 1. Geometrical parameters of the pulse tube cryocooler

Part	Length (mm)	Radius (mm)
AC	30	10
Reg	100	10
CHX	30	10
PT	120	10
HHX	30	10
Ori	15	0.5
B	50	25

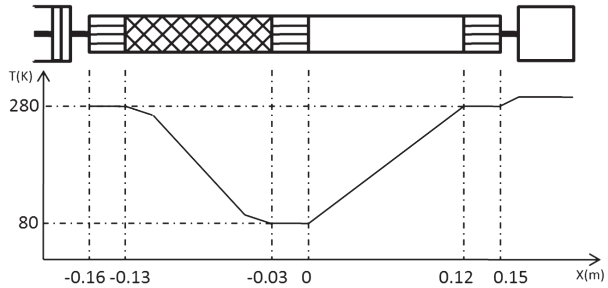

Figure 4. Temperature gradient of solid

Simulation results give T and X of any gas parcel at any time step. Temperature versus position of different gas parcel is given from Figure 5 to Figure 8.

From Figure 5, it can be known that gas parcel 1 moves only in the pulse tube in the whole cycle, and it undergoes an adiabatic process in the pulse tube. It is obvious that the first half of the cycle is different from the second half of the cycle as the temperature is different at the same position. It is because there is a phase shift between pressure and mass flow. It leads to a nonsymmetrical process in the first half and the second half of the cycle.

Figure 6 shows that gas parcel 75 moves between the PT and the CHX. The temperature of CHX is 80K, and gas parcel 75 absorbs heat from the CHX. And the slope in the CHX is lower than that in the PT, because the cross-sectional area of CHX is smaller than that of PT.

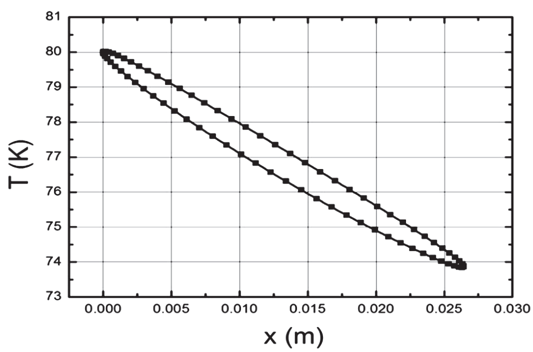

Figure 5. Temperature versus position of gas parcel 1. Gas parcel 1 enters PT(0<x<0.12) at the beginning of the first half cycle and reaches the cold end of PT at the end of the second half cycle.

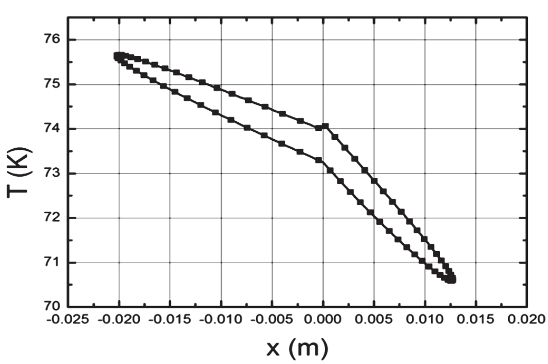

Figure 6. Temperature versus position of gas parcel 75. Gas parcel 75 enters PT at time step 75. When gas parcel 75 leaves PT, it enters CHX(-0.03<x<0).

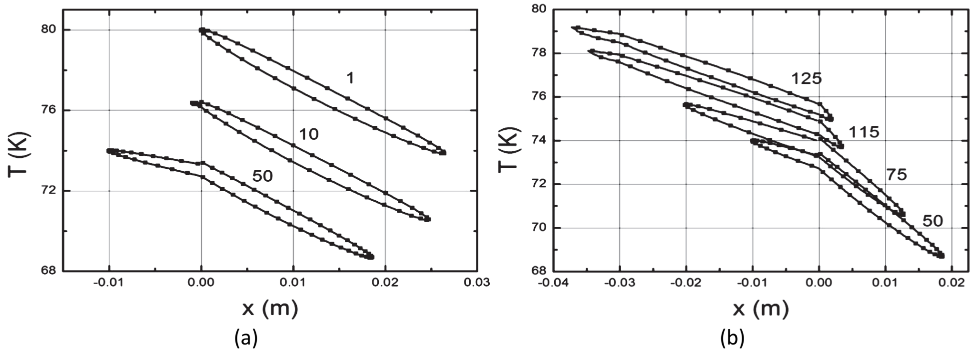

Figure 7. Temperature versus position of different gas parcels

Figure 7(a) demonstrates that the average temperature decreases from gas parcel 1 to gas parcel 50. From Figure 7(b), it can be seen that the average temperature increases from gas parcel 50 to 125. The result implies that there is a gas parcel having the lowest average temperature in a cycle.

Figure 8 shows the thermodynamic process of gas parcels in Reg. (-0.13<x<-0.03). The average temperature of a gas parcel increases from the forepart to the tail part because the temperature of a solid in the Reg. increases from the forepart to the tail part. In Figure 8(a), the temperature variation is 5.1K, while it is 6.4K in Figure 8(b) and 10.5K in Figure 8(c). So the temperature variation of the gas parcels increase from the forepart to the tail part in the Reg.

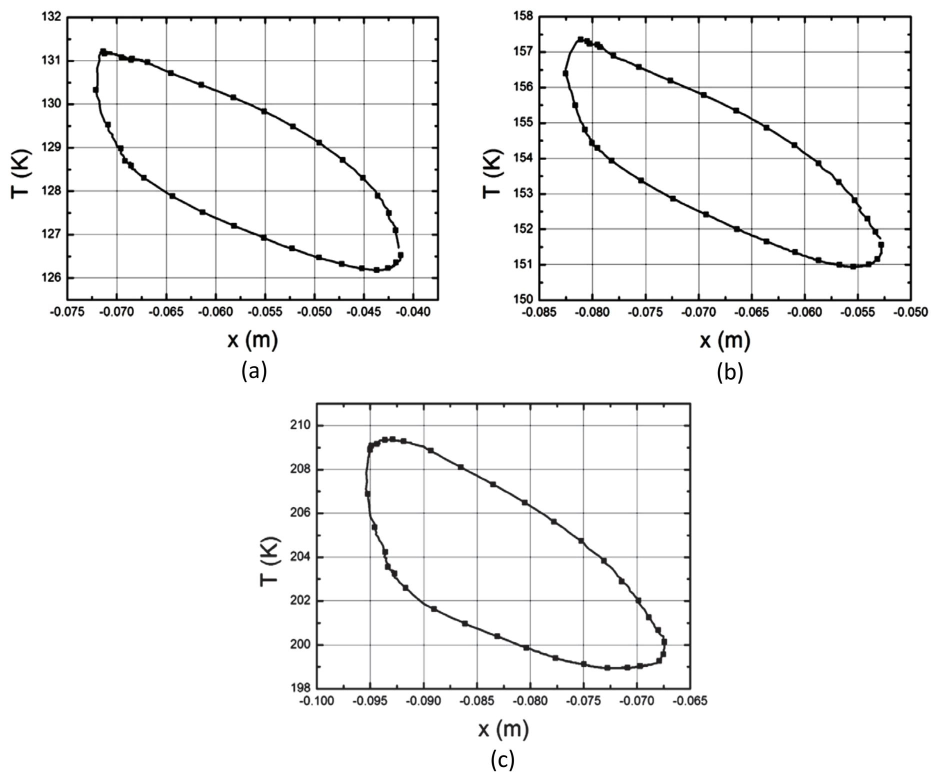

Figure 8. Temperature versus position of different gas parcels in Regenerator. 8(a),8(b) and 8(c) respectively correspond to gas parcels in the forepart, middle part and the tail part of Regenerator.

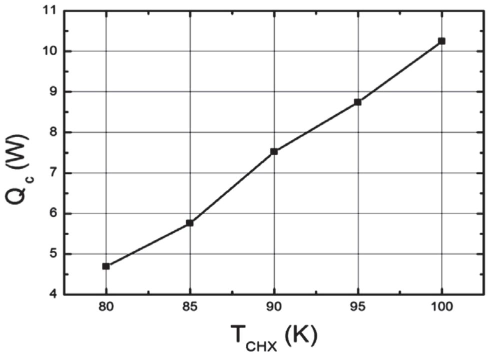

Figure 9. Refrigeration power versus temperature of CHX

From Figure 9 it can be shown that the refrigeration power Q_c increases with the temperature of the CHX. The relationship of Q_c and T_c is almost linear. The tendency is consistent with previous experimental research.

CONCLUSION

A 1-D linear type PTC model based on the Lagrange method is developed to simulate the thermodynamic cycle of gas parcels in a PTC. With this model, the temperature, the pressure and the positions of gas parcels of the whole cycle are obtained. The results show the thermodynamic processes of all of the gas parcels clearly in figures of the temperature versus position. Through the simulation results, the following conclusions can be made:

(1) The first half cycle and the second half cycle of the gas parcels in the PT are non-symmetrical since there is a phase shift between the pressure and the mass flow;
(2) Average temperature of gas parcels from 1 to n decreases firstly and then increases;
(3) The temperature variation of gas parcels increases from fore part to the tail part in Reg;
(4) Refrigeration power increases as the temperature of CHX increases, and the relationship of refrigeration power and temperature is almost linear.

The advantage of this simulation is that it follows the thermodynamic process of each gas parcel in the PTC and is helpful for studying how the gas parcels work in a whole cycle. In future work, more parameters will be taken into account, like filling pressure and frequency. More research studies will be done to determine the change in COP with different working conditions from both macro perspective and micro perspective.

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