An Exploration of a Micro-Cryocooler with a Warm-Displacer Phase Shifter

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ABSTRACT

The displacer type pulse tube cryocooler can theoretically achieve any phase angle between the pressure and volume flow oscillation and recover the expansion work at the pulse tube warm end. Significant developments with the displacer approach have demonstrated very promising improvements to the efficiency, as compared with the other phase shifting options. Extending this same phase shifting approach to a miniature pulse tube cryocooler may provide advantages over the other alternatives, since for example even inertance tubes are severely limited in their phase shifting capability when the associated acoustic power decreases below 20 W. The detailed design method associated with the displacer is shown in this work, introducing the method for choosing the major parameters of the displacer in order to obtain a desired phase angle for a micro-cryocooler. An example case displays the coupled relationship between the displacer radius and mass, piston axial length, frequency, and spring stiffness. The same parameters couple the piston displacement to the phase angle between the pressure wave and piston force. A number of 3-D parametric maps are generated relating the various key design parameters and providing the means for a rough initial displacer design.

INTRODUCTION

A numerical analysis about the warm gas driven displacer with high potential performance was presented by S.W Zhu and M. Nogawa [1] . One of the results shows that the cryocooler system has an optimal phase angle between the displacer and compressor piston in order to obtain the highest COP. Z.R. Guo and Y.Z. Lin [2] researched the displacer rod effect on the cryocooler system experimentally and found that there is an optimum displacer rod diameter for obtaining the highest efficiency when the parameters of the mass-spring-damper system associated with the displacers remains approximately constant. In 2019, C. Xi [3] designed a displacer phase shifter to replace the inertance tube, that obtained a higher efficiency. In addition, the dynamic analysis of the displacer was represented in their work. Also, several researchers investigated the displacer phase shifting capacity, as well as its effect on the system performance coupled with other operating parameters. Although some tests of the displacer type cryocooler have been carried out both experimentally and theoretically[4-6], a systematic analysis of the working mechanism has yet to be developed that could serve as an initial (approximate) design tool for various applications. Previously, displacers have been made through a combination of past experience and experimental iteration. The present work provides a design process for the displacer geometry and the coupled relationship between the

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displacer radius and mass, piston axial length, frequency, and spring stiffness. The same parameters couple the piston displacement to the phase angle between the pressure wave and piston force. As a result, several 3-D parametric maps are generated relating the various key design parameters and providing the means for a rough initial displacer design.

RESEARCH METHOD

Phase angle and displacement equations

The goal of the analysis shown below is to develop a relationship between the movement of a warm displacer and the phase angle between that motion and the associated pressure wave, so that the key design parameters of a warm displacer can be chosen in order to fabricate an effective displacer. Such a relationship involves the displacer mass and geometry as well as a gas flow damping term. After establishing the key relationship, its variation as a function of the mass and geometry factors will be graphically demonstrated.

The analysis begins by using a set of equations developed by Xi Chen's research team[3], describing the displacer displacement (X_e), and the phase angle (φ) between the pressure wave and displacer displacement. The equations were expressed as:

$$\varphi = \arctan\left(\frac{c_g\omega}{m\omega^2-k}\right),\ X_e = \frac{A_e\Delta P_e - A_{sc}\Delta P_{sc}}{\sqrt{(k-m\omega^2)^2+(c_g\omega)^2}} \tag{1}$$

Where m is the displacer mass (aluminum), A_e is the cross sectional area of the displacer (and expansion space) facing the warm end of the pulse tube, and A_{sc} is the cross sectional area of the displacer (and compression space) facing the compressor. Note that A_{sc} does not include the cross-sectional area of the displacer rod. Figure 1 shows the structure of the displacer system. Additional parameters related to the system operating conditions include the amplitude of the pressure oscillation in the displacer expansion space (ΔP_e), the amplitude of the pressure oscillation in the compression space (ΔP_{sc}), and the angular frequency ω. The term k represents the spring stiffness. The damping coefficient c_g is a special parameter associated with both the displacer geometry and the system operating conditions. The solution to equation (1) can be easily obtained by expressing c_g in terms of more fundamental parameters, which are related to the displacer geometry and the operating conditions. Fundamentally, damping is created by the drag force on the displacer. The drag force is expressed as,

$$F_D = C_f\frac{1}{2}\rho\bar{v}^2A_{gap} = \frac{\mu\bar{v}}{2X_e}A_{gap} = c_g\dot{x}_e \tag{2}$$

Where F_D is the drag force, c_g is the damping coefficient, ρ is the density of the fluid, A_{gap} is the cross-section area of the gap between the displacer and the wall, and v is the velocity of the displacer. The friction coefficient c_f is related to the Reynolds number Re defined by the gap width, the fluid

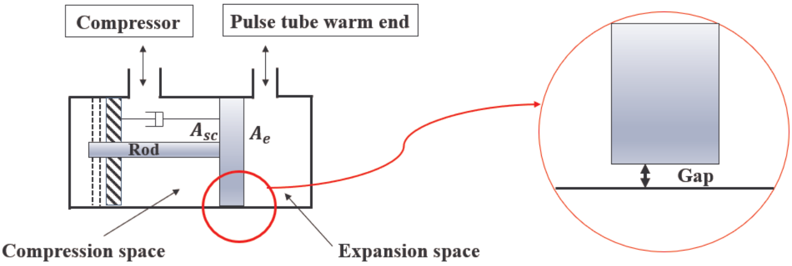

Figure 1. Geometric configuration of the warm displacer relative to the pulse tube and compressor

viscosity and the velocity of the displacer. Note that the gas damping coefficient could be expressed as $c_g = \frac{\mu A_{gap}}{2X_e}$, and $\dot{x}_e$ is the mean velocity of the displacer.

Based on the analysis above, the two equations regarding the phase angle and the displacer displacement could be solved by an iterative calculation. Such a detailed design calculation for the displacer was carried out in this work, where the major parameters of the displacer are determined in order to obtain a reasonable phase angle in the micro cryocooler system.

Example case and results

An example iteration calculation case using EES[7] for the initial displacer designation and the output figures is shown in this part.

First, the displacer radius r_{dis} and the parameter c (=mω^2 - k) are used as independent variables, and an initial displacer displacement x_e is set to start the iterative computation. After each iteration, a new value of X_e is obtained. If the value of $x_{e_{err}} = |X_e - x_e|$ is larger than 1e-7, the calculated new displacer displacement X_e will be input as the new x_e and the iteration continues until the value of $X_{e_{err}}$ is smaller than 1e-7. Subsequently, various 3-D parametric graphs are generated to display the relationships between the displacer displacement X_e, the phase angle φ, the displacer radius r_{dis} and the composite parameter c. The operating frequency $f = \omega/2\pi$ was also change in order to investigate its influence on the displacer displacement X_e, the phase angle φ and the parameter c.

Figure 2a and Figure 2b, the case with a fixed frequency of 150 Hz, demonstrate that the phase angle φ does not change with variations of c and r_{dis}. The displacement X_e is more sensitive to changes in the displacer radius r_{dis} than changes of c. Figure 2c and Figure 2d were obtained with the fixed r_{dis}=5 mm. Figure 2c gives the displacement X_e distribution with c and frequency, Figure 2d shows the phase angle φ dependence on frequency. The phase angle changes significantly with the frequency, while the displacement X_e is less sensitive to the frequency.

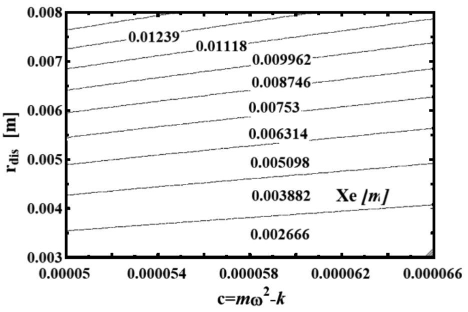

Figure 2a. X_e distribution with the c and r_{dis}

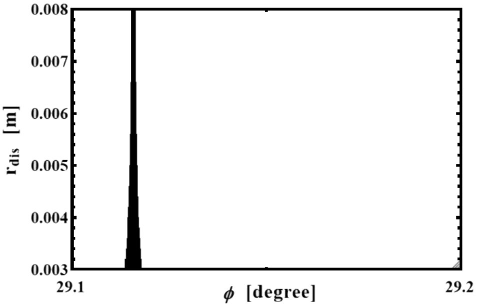

Figure 2b. φ distribution with r_{dis}

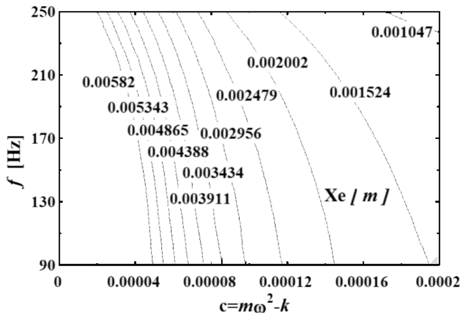

Figure 2c. X_e distribution with the c and frequency

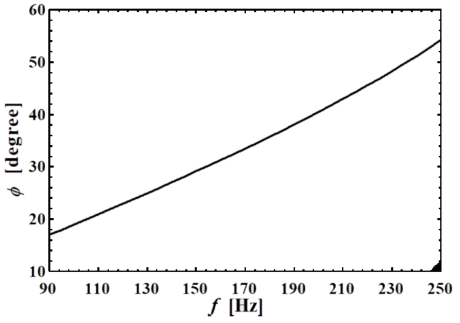

Figure 2d. φ distribution with frequency

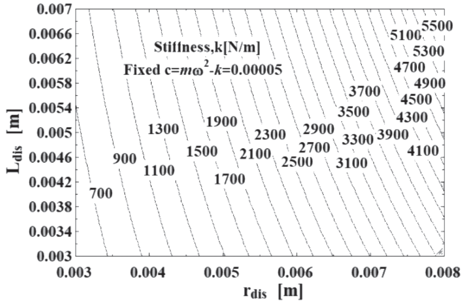

Figure 3a. Stiffness distribution with L_{dis} and r_{dis}

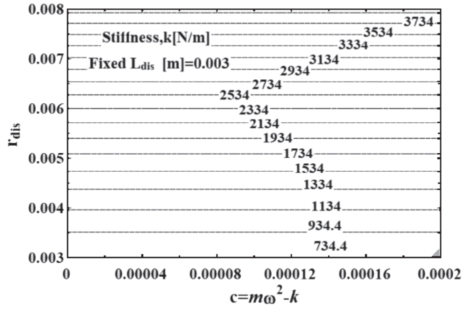

Figure 3b. Stiffness distribution with r_{dis} and c

In a second set of calculations, the displacer radius r_{dis} and the displacer length L_{dis} are used as independent variables to determine values of the spring stiffness k for a fixed frequency of 150 Hz and a constant value of c. Figure 3a displays the result for c = 0.00005 and Figure 3b displays the results for L_{dis} =0.003 when c changes. The range of c corresponds to that also used in Figure 2c, and the corresponding displacer movement of 5 – 2 mm.

This displacer design method is helpful for making a well-matched displacer of the displacer-type cryocooler. Figure 4 displays the phase relationship information for the displacer-type cryocooler system, where U_w is the volume flow rate at the warm end of the regenerator, U_c is the volume flow rate at the cold end of the regenerator , U_e is the volume flow rate at the warm end of the pulse tube, U_{eb} is the volume flow rate at the compression space facing the compressor, x_e is the displacer displacement, x_c is the compressor piston displacement, and P is the pressure wave in the system. For a desired phase angle between the pressure wave and the volume flow rate in the regenerator or in the pulse tube, the phase angle (φ) between the pressure wave and the displacer displacement X_e could be obtained by the relationship 90° - θ_{pte}, where θ_{pte} is the phase angle between the pressure (P) and the volumetric flow rate at the warm end of the pulse tube (U_e). The displacer design method above can thus be used to design a matched displacer that will result in the desired phase angle in the regenerator or in the pulse tube. Conversely, if the phase angle between the pressure wave and the displacer displacement X_e is designed, then the phase angle between the pressure wave and the volume flow rate in the pulse tube could be known by the relationship θ_{pte} = 90° - φ. In other words, a displacer can be matched to the cold finger of a pulse tube cryocooler system.

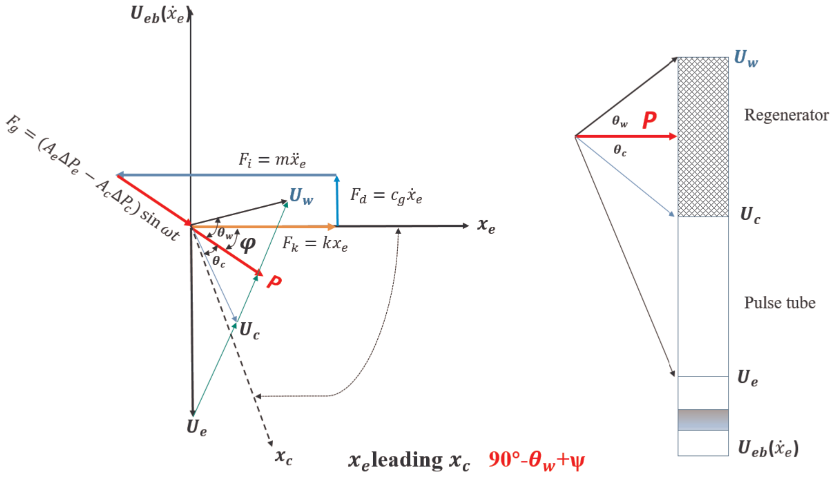

Figure 4. Phase relationship information of the displacer-type cryocooler system

Recognizing the strong relationship between the phase angle and frequency in the displacer as shown in Figure 2d, and knowing from previous reports[8] of the optimal frequency associated with the regenerator design, an overall system design process will be required to obtain peak performance in a displacer type pulse tube cryocooler.

ACKNOWLEDGMENT

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