Study on Flow Characteristics of Regenerator and its Application in a Stirling Cooler

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ABSTRACT

Recently, a Stirling cooler using a regenerator made from winded wire mesh has been tested in our lab. To guide the design of the cooler, the flow resistance through the new regenerator has to be tested. The focus of this paper is to generalize and compare different methodologies and different generated friction factors. Due to the page limits, some details of the analysis are not shown here but just the main findings. Using a Chinese idiom 抛砖引玉 (throw a brick to attract a jade), we hope our investigation may arouse others' interest and a better contribution to this topic. The main conclusion is that there is no big difference on friction factors between oscillating flow and steady flow, at least inside the regenerator normally used. The experimental results on the winded wire mesh regenerator flow resistance and cooler performance are also briefly reported. Typically, with an average charge pressure of 2.5 MPa and the winded wire mesh regenerator, the cooler can provide 102 W of cooling power at 235 K with a second law efficiency of 21.4%.

INTRODUCTION

It has long been a puzzle or the source of dispute concerning the flow resistance through the regenerator, a core component in the oscillating-flow-based thermal system such as the Stirling cryocooler, the pulse tube cryocooler, the Stirling engine and the thermoacoustic engine, etc. The underlying reason for this dispute is the flow here is oscillating flow which is theoretically more complicated than the steady flow. It is very clear that in a simple channel such as the channel formed by two parallel plates or circular tube, the viscous drag can be quite different in between the oscillating flow and steady flow due to the skin effect, i.e., the perpendicular/radial velocity distribution apparently changes as the Valensi number increases. However, it is somehow ambiguous as to how to treat the oscillating flow through the regenerator, which is typically filled with stacked mesh screens, small beads or other types of porous medium. Is it still possible for the flow to statistically develop an averaged velocity profile different from the steady flow? In addition to this ambiguousness, due to the reservoir effect through the regenerator as a result of the pressure oscillation intrinsic in these systems, the macroscopic end-to-end regenerator pressure drop can be easily confused with the friction factor which should be the fundamental one to be discussed.

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Re-igniting our recent interest in this topic is due to the fact that we are currently working on a low-cost free piston Stirling cooler (FPSC) for deep freezing applications. The thermal performance using winded wire mesh has been reported [1], which brings low material waste, simplified manufacturing process and possibly low flow resistance. To guide the design of the cooler, quantitative study of the friction factor through the regenerator should be done. To choose the methodology for the study, a literature survey has been performed and is discussed below.

REGENERATOR FLOW STUDY REVIEW, COMPARISON AND DISCUSSIONS

In 1964, Kays and London investigated the flow characteristics of porous medium in steady flow tests [2]. The friction factor f is defined in reference to Darcy-Weisbach form [3]:

$$\frac{\Delta p}{L} = -\frac{1}{2}\rho u^2 \frac{f}{d_h}, \; d_h = \frac{d_w \varepsilon}{1-\varepsilon} \tag{1}$$

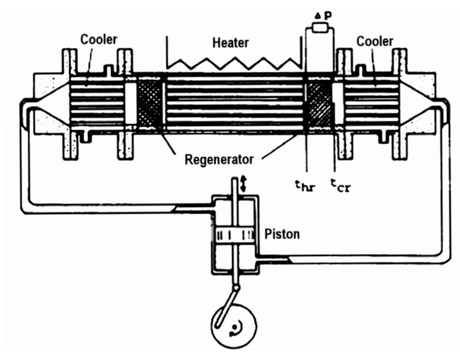
The obtained empirical formula for the stacked wire mesh (SWM) is [4]:

$$f = \frac{5072 - 14180\varepsilon + 10176\varepsilon^2}{Re} - 11.28 + 42.8\varepsilon - 34.4\varepsilon^2 \tag{2}$$

Where Δp is the pressure drop across the test section, L is the length, u is physical fluid velocity, ρ and ε are fluid density and filling porosity, respectively. d_h is the hydraulic diameter, characterizing the scale of the flow channel, and d_w is the wire diameter.

In 1990, Tanaka et al investigated the flow characteristics of stacked wire mesh and metallic sponge in oscillating flow tests, where a displacer is used to drive the gas back and forth through the regenerator [5]. In 2021, Perrella et al investigated the flow characteristics of stacked wire mesh and metallic spheres at different operation temperatures in oscillating flow tests [6]. Their setups are shown in Figure 1a and Figure 1b, respectively.

a)



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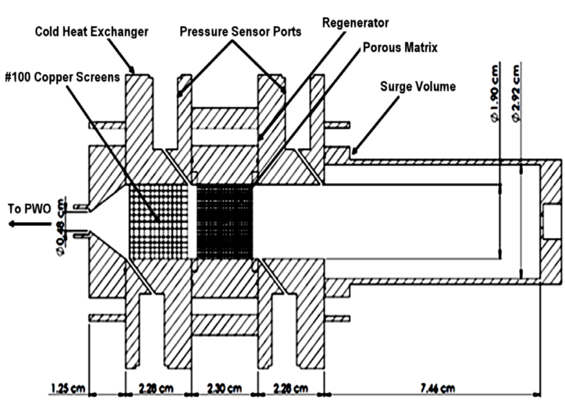

Figure 1. Schematic of pressure drop measurement setups in oscillatory flow by Tanaka et al. [5] (top) and Perrella et al. [6] (bottom).

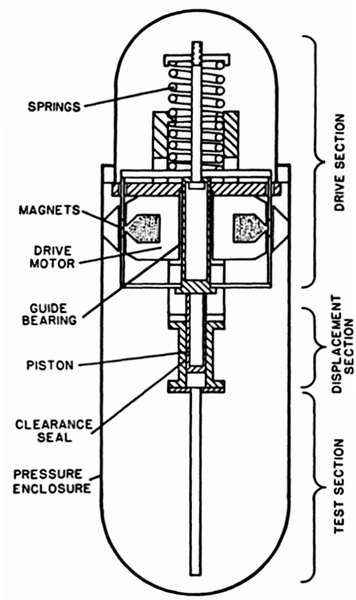

Figure 2. Schematic of friction factor measurement in oscillatory flow by Gedeon et al [7].

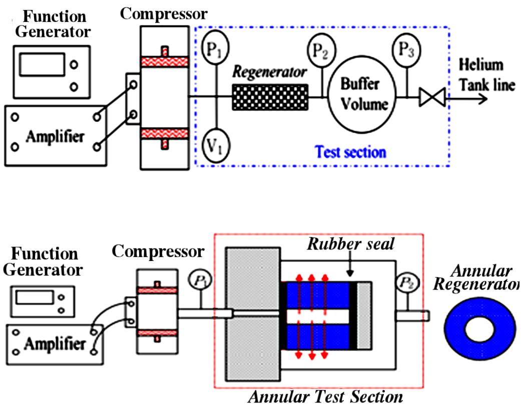

Figure 3. Schematic diagram of the setup measuring axial pressure drop (left) and radical pressure drop (right) in oscillatory flow by Cha and Clearman et al [9,10].

Using peak velocity u_{peak} and related peak pressure drop Δp_{peak} to correlate friction factor, the obtained empirical formula for SWM are:

$$f = \frac{175}{Re} + 1.60 \quad \text{(Tanaka)} \tag{3}$$

$$f = \frac{156}{Re} + 1.33 \quad \text{(Perrella)} \tag{4}$$

In 1996, Gedeon et al. investigated the flow characteristics of SWM and metallic felt in oscillating flow tests [7] shown in Fig. 2.

Quoting [7], the friction factor is obtained using the relationship with cycle-integrated pumping power unit void volume w_r:

$$w_r = \left\langle \frac{\partial p}{\partial x} u \right\rangle = -\frac{1}{2d_h} \left\langle f(Re) \rho u^2 |u| \right\rangle \tag{5}$$

The friction factor for SWM, takes the form from so-called modified Ergun equations (section 23.1 Woven Screen Matrix in [8]):

$$f = \frac{129}{Re} + 2.91 Re^{-0.103} \tag{6}$$

where Reynolds number is calculated using instantaneous velocity and hydraulic diameter.

From 2007, Cha and Clearman et al. have investigated the axial and radial flow characteristics of the stacked wire mesh in steady and oscillating flow (Fig. 3) by combing experiments and Fluent [9,10]. The basic methodology is that by adjusting coefficients such as the permeability in the simulation, when the numeric simulation generates pressure drop close to the experimental results, the coefficients will be fixed and used to calculate the conventional friction factor.

The momentum source term in Fluent can be expressed as [11]:

$$\vec{\boldsymbol{S}}_i = \frac{\varepsilon^2 \mu \vec{\boldsymbol{v}}}{\alpha} + \frac{\varepsilon^3 C_2}{2} \rho \vec{\boldsymbol{v}} |\vec{\boldsymbol{v}}| \tag{7}$$

Table 1. Comparison of different formulas (400# stacked wire mesh).

	f - Re	20	40	60	80	100	150	200
Original Cha's [9]	f=257/Re+6.85	19.7	13.3	11.1	10.1	9.4	8.6	8.1
Modified Cha's	f=123/Re+2.27	8.4	5.3	4.3	3.8	3.5	3.1	2.9
Kays et al. 's	f=132/Re+1.86	8.5	5.2	4.1	3.5	3.2	2.7	2.5

where $1/\alpha$ and C_2 is the inverse permeability and inertial resistance factor, respectively. While in Cha's methodology, the source term is written as:

$$\frac{\mu}{\beta}u+\frac{C\rho}{2}|\vec{u}|u=\frac{1}{2}\rho|\vec{u}|u\frac{f}{\beta^{0.5}} \tag{8}$$

The relationship between the above coefficients and the coefficients in Fluent is:

$$\beta=\frac{\alpha}{\varepsilon^2},\quad C=\varepsilon^3 C_2 \tag{9}$$

However, we do have a concern that when making the changes, Cha et al may have lost the inverse porosity squared and the porosity cubic respectively in dealing with above coefficients. For example, the empirical formula with $\beta^{0.5}$ as channel characteristic scale for 400# mesh in Cha's work is:

$$f=\frac{2}{Re_\beta}+0.60,\quad Re_\beta=\frac{\rho|\vec{u}|\beta^{0.5}}{\mu} \tag{10}$$

For comparison, a transformation has been done on the Cha's results by introducing the inverse porosity squared and the porosity cubic into the formula (modified Cha's in Table 1) and re-write it in a more conventional friction factor formula. Table 1 shows the comparison of the friction factor calculated by Cha's, modified Cha's and Kays et al. empirical formula. The largest difference between modified Cha and Kays & London is within 14%.

One more puzzle with the comparison between oscillating flow and steady flow in Fig. 17 of [9] is that, the conclusion therein claims that that there is a big difference of friction factor values between those obtained from steady flow experiments and oscillating flow experiments, to which we still do not have answers.

Bearing these results in mind, in the following we will discuss the philosophies behind these measurements and how they have been used in some professional simulation software. We start with a regular channel.

In an idealized parallel plate channel (with infinite depth), the skin effect will cause a different radial velocity distribution, which depends on the frequency. Figure 4 shows a typical velocity distribution at a certain time in a cycle. With small perturbation and laminar flow assumptions, the momentum equation can be simplified using complex notation according to thermoacoustic theory [12].

$$i\omega\rho_m\hat{u}_1=-\frac{d\hat{p}_1}{dx}+\mu\frac{\partial^2\hat{u}_1}{\partial y^2} \tag{11}$$

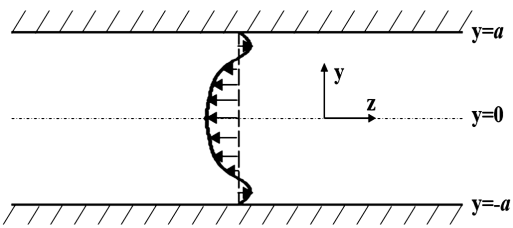

Figure 4. Instantaneous axial velocity distribution between parallel plates in oscillating flow.

With non-slip boundary condition, the solution of Eq.11 leads to:

$$\hat{u}_1 = \frac{i}{\omega\rho_m}\left[1-\hat{h}_\mu(y)\right]\frac{d\hat{p}_1}{dx},\ \hat{h}_\mu(y) = \frac{cosh\left[(1+i)y/\delta_\mu\right]}{cosh\left((1+i)a/\delta_\mu\right)} \tag{12}$$

Substituting this solution back into second term of the right-hand side of Eq.11, the relationship between viscosity related pressure gradient $\Phi\left(\langle\hat{u}_1\rangle\right)$ and area-weighted average velocity can be expressed as:

$$\Phi\left(\langle\hat{u}_1\rangle\right) = \frac{i\omega\rho_m(\hat{F}_\mu - 1)}{\hat{F}_\mu}\langle\hat{u}_1\rangle \tag{13}$$

where F_μ is related to Valensi number, Va.

$$\hat{F}_\mu = 1 - \frac{4}{\sqrt{iVa}}tanh(\frac{\sqrt{iVa}}{4}),\ Va = 2\left(\frac{d_h}{\delta_\mu}\right)^2 \tag{14}$$

In reference to Eq.1, the pressure gradient can be expressed using an effective friction factor f_{eff}:

$$\Phi\left(\langle\hat{u}_1\rangle\right) = -\frac{1}{2}\frac{\rho_m\langle\hat{u}_1\rangle\left|\langle\hat{u}_1\rangle\right|}{d_h}f_{eff},\ f_{eff} = \frac{\hat{s}_1}{Re_1},\ \hat{s}_1 = 2Va\frac{\hat{F}_\mu - 1}{i\hat{F}_\mu} \tag{15}$$

where Re_1 is peak Reynolds number.

Figure 5 shows how s_1, real part, imaginary part and its phase, changes with Valensi number. At an extreme condition where Va is very small, it can be clearly seen that the real part reduces to 96 and imaginary part is almost zero, which means the friction factor is the same as that in a steady laminar flow. As Valensi number increases, real part slightly increases while the imaginary part quickly increases, which brings an apparent phase angle of s_1. The means that in a bigger channel relative to the viscous penetration depth, the pressure drop will have an apparent phase difference with the velocity, different from that in a steady flow.

In an ordinary regenerator, sometimes referred to as the porous medium, the Valensi number is quite small which leads to a natural thought that, even with an oscillating flow, the friction factor may take the same form as in a steady flow. Actually, among different simulation software such as Regen, DeltaEC and Sage, different practices have been used. This is the focus of the following discussions.

Regen [13], uses data from Kays and London [2]

In Regen, which is a software from NIST to calculate regenerator (simple tube can also be calculated), Section 5.

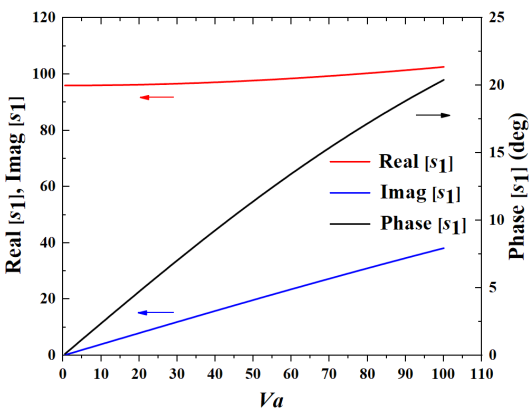

Figure 5. Variation of $\hat{s}_1$ with Va between parallel plates in oscillating flow.

The following momentum equation has been used in Regen.

$$\frac{\partial p}{\partial x} - f\left(\rho, T, u\right) = 0 \tag{16}$$

It is claimed that the data is from Kays and London and it is also mentioned that since the data was obtained from steady flow and may not be accurate for oscillating flow in a regenerator. It is somewhat interesting to see that in Eq.16 the time derivative disappeared. Actually, the inertial effect, related to the time derivative for the velocity is too small, which is also supported by thermoacoustic theory.

DeltaEC [14], uses data from integrated Kays and London [2]

With so-called Iguchi assumption [15], the flow at any instant of time has no memory of its recent history ($|\xi_1| \gg r_h$), which means the instantaneous pressure gradient should be related to the instantaneous velocity in the same way that in steady flow.

Pressure gradient from viscous effect by referring to Darcy-Weisbach formula is written as:

$$\Phi(x,t) = -\frac{1}{2}\rho(t)\left|\langle u(t)\rangle\right|\langle u(t)\rangle\frac{f(t)}{r_h}, \; f \simeq \frac{c_1(\varepsilon)}{Re} + c_2(\varepsilon) \tag{17}$$

Through integration over one cycle and using complex notation, the complex pressure gradient is written as:

$$\frac{d\hat{p}_1}{dx} = -i\omega\rho_m\left[1 + \frac{(1-\varepsilon)^2}{2(2\varepsilon-1)}\right]\langle\hat{u}_1\rangle - \frac{\mu_m}{r_h^2}\langle\hat{u}_1\rangle\left[\frac{c_1(\varepsilon)}{8} + \frac{c_2(\varepsilon)Re_1}{3\pi}\right] \tag{18}$$

An effective friction factor is thus defined as:

$$f_{eff} \simeq \frac{c_1(\varepsilon)}{Re_1} + 8c_2(\varepsilon)/3\pi \tag{19}$$

where Reynolds number is defined as:

$$Re_1 = \frac{\rho_m\left|\langle\hat{u}_1\rangle\right|d_h}{\mu_m} \tag{20}$$

However, in Swift's theoretical analysis [4], they directly compare the integrated friction factor with experimental results obtained by Tanaka et al, with friction factor correlated with the peak Reynolds, which is not appropriate. Meanwhile, it is interesting to notice that $8/3\pi$ is very close to 1 and if c_1/Re occupies a major fraction of the friction factor, the impact on the prediction of the system performance is not big.

Sage [8], uses its embedded formula

As mentioned in the first section, Sage uses the friction factor obtained from their oscillating flow experiments [7]. The momentum equation takes the form

$$\frac{\partial\rho uA}{\partial t} + \frac{\partial\rho uuA}{\partial x} + \frac{\partial p}{\partial x} + FA = 0 \tag{21}$$

where F may be formulated in terms of Darcy friction factor *f* and a total local loss coefficient K:

$$F = -\left(\frac{f}{d_h} + \frac{K}{L}\right)\frac{\rho u|u|}{2} \tag{22}$$

Interestingly, *f* here is essentially an integrated effect from oscillation flow experiments and is loaded into time-dependent equations. The practice works quite well in terms of predicting cooler performance.

Table 2. Comparison with different formulas using Sage as the test platform, a 77 K pulse tube cooler is simulated with a fixed pressure wave amplitude in the compression space, mesh 400# , porosity 73.6%.

	f - Re	$p_{c,1}$	$\triangle p_1$	Q_c	W_a	$\eta_{r,e}$
Kays et al	f=148/Re+1.59	3.5bar	23.6kPa	13.2W	178.5W	15.5%
Gedeon et al	f=129/Re+2.91$Re^{-0.103}$	3.5bar	23.5kPa	13.2W	178.5W	15.5%
Tanaka et al	f=175/Re+1.60	3.5bar	26.3kPa	12.9W	178.2W	15.3%
Perrella et al	f=156/Re+1.33	3.5bar	23.2kPa	13.2W	178.5W	15.5%

Note: $p_{c,1}$ is the amplitude of first-order pressure wave in the compression space, Q_c and W_a is the cooling capacity and input acoustic power, respectively. $\eta_{r,e}$ is the second law efficiency in terms of electrical power consumed.

To some extent, this may not be so surprising if considering the above discussions about Eq.17 and 19, which shows a small change on the formula coefficients after the integration over one cycle.

Comparison between different formulas using Sage as the common platform

To further show the effect of using different formulas, an inertance type pulse tube cryocooler (Tc=77K/Th=296K) under development is simulated using Sage software. The dynamics as well as thermal performance has been compared in Table 2. The simulation results show that the largest difference in table values is within 14%.

EXPERIMENTAL SETUP AND RESULTS

Regenerator Flow Resistance Measurements

Due to page limit, the data processing methods will not be introduced here. Readers can refer to [6] for a similar approach. The key consideration for the setup is that, the area-weighted average velocity at the regenerator section is calculated from the pressure oscillating in the right-side void volume with an adiabatic compression assumption, and the regenerator section is short enough to avoid its reservoir effect. Typically, the largest difference between the velocities at the right and left side of the regenerator is within 6%. Meanwhile, the pressure drop is almost in phase with the velocity.

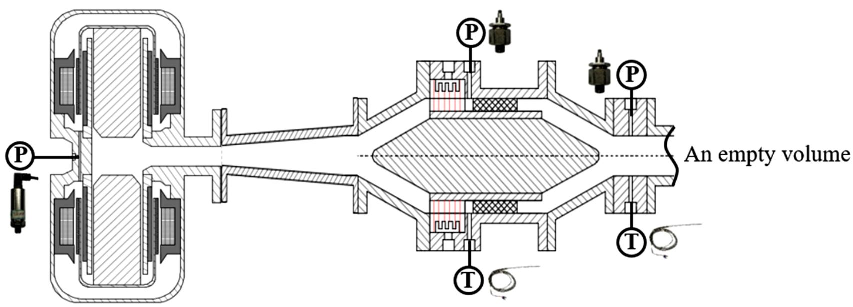

Figure 6. Schematic of the setup for regenerator flow resistance measurements, typically using Helium gas, 2.5 MPa mean pressure, 75 Hz, ambient temperature 298 K.

(a) Stacked wire mesh (SWM) (b) Winded wire mesh(WWM)

Figure 7. Photos of two regenerator fillers to be tested.

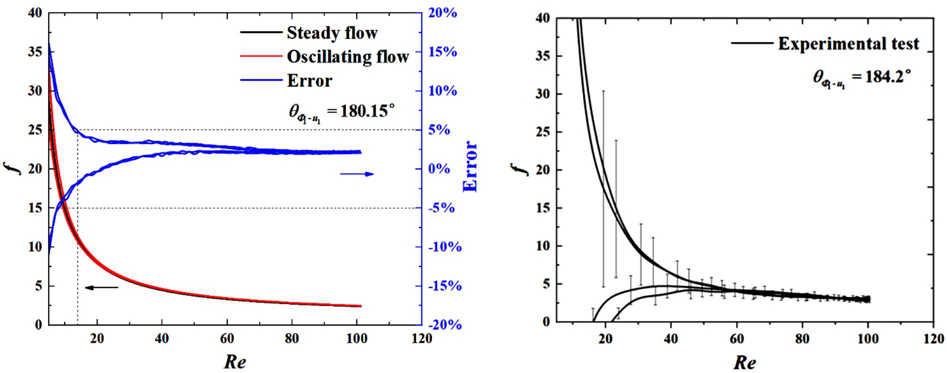

Figure 8. Comparison of friction factor between steady flow and oscillating flow in the CFD simulation and experiments.

At this step, it is natural to ask the question, given Iguchi assumption previously mentioned and the deduction philosophy, that why we do not use the different velocity values (Reynolds number) in just one cycle to correlate a friction factor. Indeed, we have checked this practice through both CFD simulation (drawing 3D details of the wire mesh and the gas space and using a direct simulation) and experiments. In the left figure of Figure 8, we first calculate the friction factor under steady flow using different velocity values. Then we calculate different friction factors in an oscillating flow in one cycle whose peak velocity is the biggest one used in the steady flow. In this way, we can correlate friction factor in just one cycle. It can be seen from the left figure that there is almost no difference in between the steady flow and the oscillating flow. However, in the experimental results, there is an apparent phase difference of 184.2° between pressure drop across the regenerator and the velocity. Using the data in one cycle (for an absolute velocity value, we normally encounter four times in one cycle except for the nodes and anti-nodes) actually lead to hysteresis in the f curve, especially when the Reynolds number is below 60 shown in the right figure. For this reason, in the following experiments, we just use peak Reynolds number to correlate friction factor based on lots of cycles with different peak velocities.

In this manner, a typical friction factor for SWM (180#) obtained by peak-to-peak value is:

$$f=\frac{164}{Re}+1.84 \tag{23}$$

Which is in good agreement with the empirical formula of Tanaka et al., and verifies the accuracy of the test method and data processing method of the test setup.

For the WWM, Figure 9 shows typical friction factors. With the increase of porosity, the friction factor of the WWM decreases. The friction factor of the WWM is about 10% lower than that of the SWM under the same porosity. The final empirical formula for WWM (180#) takes the form:

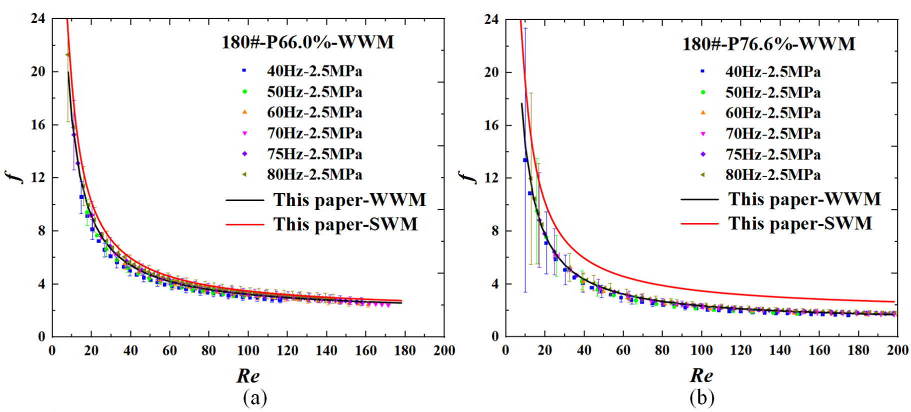

Figure 9. The resistance coefficient of the 180# WWM filler at different porosities.

$$f = \frac{-179 + 1018\varepsilon - 797\varepsilon^2}{Re} - 23.9 + 78.5\varepsilon - 60.0\varepsilon^2 \tag{24}$$

The following briefly introduces FPSC performance. For more details, readers can refer to our previously published work [1]. The FPSC is composed of a linear compressor, a hot-end heat exchanger, a regenerator, a cold-end heat exchanger and a displacer. The cooler is driven by a linear compressor, and the displacer connecting rod passes through the drive piston and is supported by a planar spring in the compressor's back space. Two identical folded fin heat exchangers are used in the cooler. Typically, FPSC works with a mean pressure of 2.5 MPa helium gas, an operating frequency of 73 Hz, the ambient temperature of 298 K. Coefficient of performance (COP) is used to evaluate the cooling performance of the cooler and is defined as the cooling capacity divided by the input electrical power.

Typically, the cooler can provide 102 W of cooling power at 235 K with a second law efficiency of 21.4% when the porosity is 72.7%. The performance is mediocre compared with that using stacked mesh. Meanwhile, the difference between experimental and simulated COP values is about 30%. The possible reasons are flow mal-distribution and mechanical losses, etc, which needs to be studied.

CONCLUSIONS

The main focus of this paper is to briefly review and discuss different methodologies used to determine the friction factor in an oscillating flow regenerator. Winded wire mesh regenerator has also been tested and used in a Stirling cooler with a mediocre performance. Sources of losses remain to be found. Most important conclusions are as follows.

Within the Reynolds range and frequency range of a cryocooler, there seems to be no possibility for the flow to develop unusually different distribution from steady flow while Cha's findings remain to be explained. Meanwhile, the inertial part in the momentum equation, i.e., the acceleration part (time derivative of velocity) can be omitted with negligible influence.

The thermal performance predictions based on different friction factors used in Regen, DeltaEC, Sage, or from Tanaka and Perrella are not so different among them, as evidenced through calculations based on a common Sage platform. This further prove that there may not be big difference on friction factors between oscillating flow and steady flow, at least inside the regenerator normally used. Meanwhile, the heat transfer study inside the porous medium remains be done in the future.

ACKNOWLEDGMENT

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REFERENCES

1. Cui, Y.H., Qiao, J.X., Song, B., Wang, X.T., Yang, Z.h., Li, H.B., Dai, W.., "Experimental study of a free piston Stirling cooler with wound wire mesh regenerator," *Energy*, vol. 234 (2021), pp. 121287.
2. Kays, W.M. and London, A.L., *Compact Heat Exchangers*, 2nd Edition, McGRAW-HILL, Inc (1964).
3. Nield, D.A., Bejan, A., *Convection in Porous Media*, 2nd Edition, Springer, New York (1999).
4. Swift, G.W., Ward, W.C., "Simple harmonic analysis of regenerators," *Journal of Thermophysics and Heat Transfer*, Vol. 10, No. 4 (1996), pp. 652-662.
5. Tanaka, M., Yamashita, I. and Chisaka, F., "Flow and Heat Transfer Characteristics of the Stirling Engine Regenerator in an Oscillating Flow," *JSME International Journal*, Series a!, Vol. 33, Vo.2 (1990), pp. 283-289.
6. Perrella, M., Ghiaasiaan, S.M., "Hydrodynamic resistance parameters of regenerator filler materials at cryogenic temperatures," *Cryogenics*, Vol. 117 (2021), pp.103320.
7. Gedeon, D., Wood J.G., *Oscillating-Flow Regenerator Test Rig: Hardware and Theory with Derived Correlations for Screens and Felts*, NASA Contractor Report 198442 (1996).

8. Gedeon, D., *Sage version 11: user's manual*, Gedeon Associates (2011).

9. Cha, J.S, Ghiaasiaan, S.M, Kirkconnell, C.S., "Oscillatory flow in microporous media applied in pulse-tube and Stirling-cycle cryocooler regenerators," *Experimental Thermal & Fluid Science*, Vol. 32, No. 6 (2008), pp.1264-1278.

10. Clearman, W.M., *Measurement and Correlation of Directional Permeability and Forchheimer's Inertial Coefficient of Micro Porous Structures Used in Pulse Tube Cryocoolers*, Georgia Institute of Technology (2007).

11. ANSYS, INC., *ANSYS FLUENT User's Guide*, Published in the USA (2019).

12. Swift, G.W., *Thermoacoustics: A unifying perspective for some engines and refrigerators*, Acoustical Society of America Publications, Los Alamos (1999).

13. Gary, J., Abbie, O., *Regen3.3 user manual*, National Institute of Standards and Technology, Boulder (2008).

14. Ward, B., Clark, J., Swift, G.W., *Design environment for low-amplitude thermoacoustic energy conversion, DeltaEC Version 6.4 b2 Users Guid*, Los Alamos National Laboratory Network (2016).

15. Iguchi, M., Ohmi, M., Maegawa K., "Analysis of free oscillating flow in a U-shaped tube," *Bulletin of JSME*, Vol. 25, Vo. 207 (1982), pp.1398-1405.