Modeling Thermodynamic Response of a Cryocooler after Switching It Off

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ABSTRACT

We report a simple method for predicting the thermodynamic behavior of a cryocooler as it is switched off. This, for instance, is relevant in the case of sensitive devices of which, because of interference, the cooler has to be switched off when operating the device. In our method, we model each of the cooler cold stages as a thermal capacitance, and the individual temperature stages are thermally linked through conductances. The model parameters are determined from two warm-up recordings of a specific cryocooler with different heat loads. The model and method are presented and illustrated for the case of a single-stage pulse tube cooler.

INTRODUCTION

The modeling method presented in this paper is part of a project in which a cryogenic test setup will be designed and realized for measuring thermal noise of mirror coatings that are developed to be applied in the Einstein Telescope. The Einstein Telescope (ET) is a proposed underground infrastructure to host a third-generation, gravitational-wave observatory. [1] [2] It builds on the success of current, second-generation laser-interferometric detectors Advanced Virgo and Advanced LIGO, whose breakthrough discoveries of merging black holes and neutron stars over the past 5 years have ushered scientists into the new era of gravitational-wave astronomy. The Einstein Telescope will achieve a greatly improved sensitivity by increasing the size of the interferometer from the 3 km arm length of the Virgo detector to 10 km, and by implementing a series of new technologies. One of these technologies is the cryogenic cooling of the mirrors to a temperature in the range of 10 – 20 K. This will reduce the thermal noise originating from the coating of the mirrors. These mirror coatings are not readily available; they need to be developed and selected. Candidate mirror coatings will be selected by measuring the thermal noise of the coating in a Mirror Test Setup. The cryogenic design of this Mirror Test Setup is carried out by the University of Twente.

Central in the cryogenic design are a Thermal Storage Unit (TSU) and a cryocooler. We switch off the cryocooler during a measurement of mirror coating thermal noise, such that vibrations do not interfere with the measurement. The TSU is used to keep the optical bench, which is used to determine the mirror coating thermal noise, at a stable set-point temperature.

An important design aspect is the heat leak through the cryocooler in its off-state. This is a parasitic heat leak onto the TSU, therefore limiting the measurement time. In order to design the system, we therefore need to be able to dynamically model the heat flows through the cryocooler as it is switched off. For that purpose, we developed a method in which the cold stages each are considered as a thermal capacitance, with the stages thermally linked through conductances. In this paper,

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we present and discuss our method considering a single-stage cooler. In the ET mirror project a two-stage cooler will be used to which our method can be easily adapted.

In our approach, no a priori information on cooler geometry, dimensions and/or materials are required. Basically, the cooler can be considered as a black box. However, the specific cooler that is to be characterized needs to be available for measuring the cold stage temperatures as a function of time in two warm-up experiments, with different heat loads on the cold stage(s).

In the next section, the basic equations representing the model for a single-stage cooler are presented. The method is discussed that we used to determine the characteristic thermal parameters through two warm-up experiments. The method will then be illustrated for the case of a single-stage pulse tube cooler. The paper concludes with a brief summary of the presented method, and a discussion on its applications.

MODEL AND METHOD

In our approach, a single-stage cryocooler is modeled as a simple first-order thermal RC-network with the cold stage as a heat capacity C that is thermally linked to ambient via a thermal resistance R, in which both R and C are considered temperature dependent. Because the expressions in the equations of this section are simpler and clearer to derive, we prefer using the thermal conductance K defined as $K = 1/R$.

For a single-stage cryocooler, we have the following differential equation for the temperature of the cold stage in the off-state of the cooler:

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \frac{\dot{Q}_{0,1}(T_0, T_1) + \dot{Q}_{\mathrm{app}}(T_1) + \dot{Q}_{\mathrm{par}}(T_0, T_1)}{C(T_1)} \quad (1)$$

Here, T_0 and T_1 are the ambient temperature and that of the cold stage, respectively. The ambient temperature T_0 is assumed not to vary and is, therefore, a parameter (not a variable). $\dot{Q}_{0,1}$ is the conductive heat transfer from ambient to the cold stage, $\dot{Q}_{\mathrm{app}}$ is heat applied to the cold stage and $\dot{Q}_{\mathrm{par}}$ is the total parasitic heat load due to wiring, radiation, etc. C is the temperature-dependent heat capacity of the cold stage. In Eq. 1, the applied heat load $\dot{Q}_{\mathrm{app}}$ is generalized as a function of T_1, but may very well be constant.

The heat capacity C can be determined (as a function of temperature) from the difference between two warm-up recordings. This is done by first performing a 'reference' warm-up for which we denote the measured temperatures $T^0(t)$. Subsequent 'comparative' measurements are then performed, denoting temperatures $T'(i)$. In the reference measurement, we let the cooler warm up without applying any heat load; $\dot{Q}^0_{\mathrm{app}} = 0\,\mathrm{W}$ (this may be chosen nonzero as well). In comparative measurements, we choose $\dot{Q}'_{\mathrm{app}} \neq \dot{Q}^0_{\mathrm{app}}$. This will result in a different $\mathrm{d}T_1/\mathrm{d}t$ as expressed in Eq. 1.

The difference between the measurements can be expressed as:

$$\frac{\mathrm{d}T_1{}'}{\mathrm{d}t} - \frac{\mathrm{d}T_1{}^0}{\mathrm{d}t} = \frac{\dot{Q}'_{0,1}(T_0, T_1) - \dot{Q}^0_{0,1}(T_0, T_1) + \dot{Q}'_{\mathrm{app}}(T_1) - \dot{Q}^0_{\mathrm{app}}(T_1) + \dot{Q}'_{\mathrm{par}}(T_0, T_1) - \dot{Q}^0_{\mathrm{par}}(T_0, T_1)}{C_1(T_1)} \quad (2)$$

It is important to regard all variables as functions of T_1. Since the ambient temperature T_0 is assumed to be constant, the conductive and parasitic heat flows in Eq. 2 cancel as a function of T_1. Thus, we obtain:

$$\frac{\mathrm{d}T_1{}'}{\mathrm{d}t} - \frac{\mathrm{d}T_1{}^0}{\mathrm{d}t} = \frac{\dot{Q}'_{\mathrm{app}}(T_1) - \dot{Q}^0_{\mathrm{app}}(T_1)}{C(T_1)} \quad (3)$$

from which the heat capacity follows as

$$C(T_1) = \frac{\dot{Q}'_{\mathrm{app}}(T_1) - \dot{Q}^0_{\mathrm{app}}(T_1)}{\frac{\mathrm{d}T_1{}'}{\mathrm{d}t} - \frac{\mathrm{d}T_1{}^0}{\mathrm{d}t}} \quad (4)$$

These variables, and therefore C, can be determined experimentally. Eq. 4 gives the heat capacity C as a function of temperature in the measurement range of T_1.

Next, the thermal conductance K is determined from the conductive heat flow $\dot{Q}_{0,1}$ as

$$\dot{Q}_{0,1}(T_0, T_1) = G\int_{T_1}^{T_0} \lambda(T)\mathrm{d}T = \int_{T_1}^{T_0} K(T)\mathrm{d}T \quad (5)$$

The second term in Eq. 5 contains the well-known thermal conductivity integral and a geometry factor that is comprised of the effective cross-sectional area and length of the thermal connection between cold stage and ambient. Eq. 5 can be extended with $\bar{K}$ being the primitive of the function K:

$$\dot{Q}_{0,1}(T_0, T_1) = \int_{T_1}^{T_0} K(T)\mathrm{d}T = \bar{K}(T_0) - \bar{K}(T_1) \tag{6}$$

By taking the derivative of $\dot{Q}_{0,1}$ with respect to T_1 we find:

$$\frac{\mathrm{d}\dot{Q}_{0,1}(T_0, T_1)}{\mathrm{d}T_1} = \frac{\mathrm{d}}{\mathrm{d}T_1}\bar{K}(T_0) - \frac{\mathrm{d}}{\mathrm{d}T_1}\bar{K}(T_1) = 0 - K(T_1) \tag{7}$$

Here, the first term on the right-hand side is zero because T_0 is constant. As a result, we can write:

$$K(T_1) = -\frac{\mathrm{d}\dot{Q}_{0,1}(T_0, T_1)}{\mathrm{d}T_1} \tag{8}$$

The heat flow $\dot{Q}_{0,1}$ can be derived directly from Eq. 1 as:

$$\dot{Q}_{0,1}(T_0, T_1) = C(T_1)\frac{\mathrm{d}T_1}{\mathrm{d}t}(T_1) - \dot{Q}_{\mathrm{app}}(T_1) - \dot{Q}_{\mathrm{par}}(T_0, T_1) \tag{9}$$

In order to determine $\dot{Q}_{0,1}$ using Eq. 9 the parasitic heat load in the specific experimental setup needs to be estimated. Then, the conductance as a function of temperature in the measurement range of T_1 is given by Eq. 8.

ILLUSTRATION: SINGLE STAGE PULSE TUBE CRYOCOOLER

As an illustration of our method, we use data obtained from experiments that were performed in our lab on a single-stage Stirling-type pulse tube cryocooler, a Thales Cryogenics LPT9310. [3] In the experiment, the cold tip of the cryocooler was facing down and wrapped in MLI. The tip was equipped with a temperature sensor and a heater. After cooling down, the cryocooler was switched off. In two warm-up experiments, constant heat loads were applied to the cold tip of 0 and 2 W. In processing these data, we consider the 0 W measurement our reference measurement, with the 2 W data set being the comparative measurement.

The cold tip temperatures of the reference and comparative measurements are given as a function of time in Fig. 1a. In order to determine the heat capacity C, we need the derivative of the cold-tip temperature T_1 with respect to time, and more specifically that derivative as a function of T_1 (see Eq. 4). The temperature-time derivatives as function of time and temperature T_1 are given in Figs. 1b and 2a, respectively. In Fig. 2a we see that the starting temperatures are not the same as may be expected, because of the 2 W heat load in the comparative measurement. Fig. 2a also clearly shows that there is a temperature profile build-up phase just after switching off the cryocooler. The development of the temperature profile causes the rise in dT/dt which is followed by a gradual decrease up until 273 K. For a good result, the build-up phase should not be included in the evaluation of the het capacity. We therefore choose to omit temperature data below 75 K. As expressed in Eq. 4,

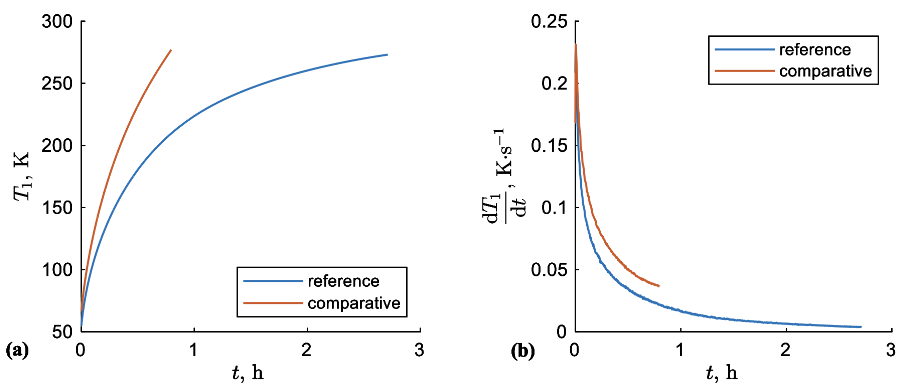

Figure 1. Method demonstrated on single-stage pulse tube cooler (Thales LPT9310); reference: 0 W heat load; comparative: 2 W heat load; (a) warm-up curves; (b) temperature-time derivatives as a function of time.

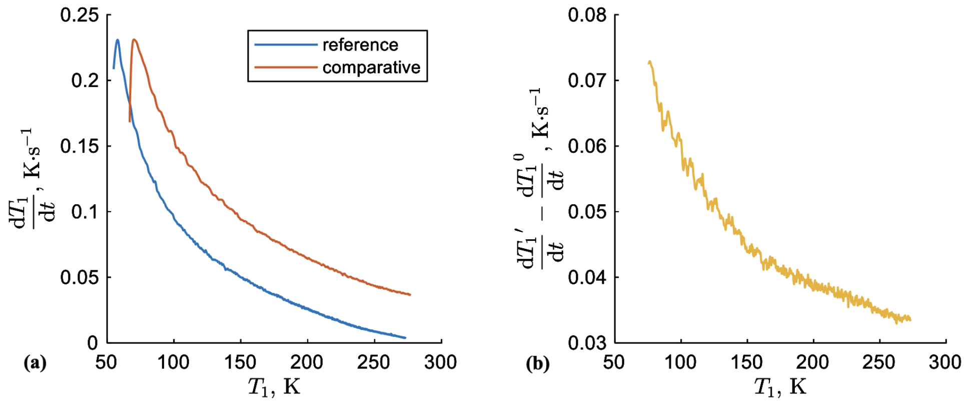

Figure 2. Method demonstrated on single-stage pulse tube cooler (Thales LPT9310); reference: 0 W heat load; comparative: 2 W heat load; (a) temperature-time derivatives as a function of cold-tip temperature; (b) difference between the curves in (a).

the next step in our method is to determine the difference in the temperature-time derivatives as function of T_1 between the comparative and reference measurements. This can only be done if the data points are recorded at the same cold tip temperatures. Since these are separate measurements, where we measure as a function of time, this is not the case. Therefore, a linear interpolation is used for $dT/dt\,(T)$. We can then evaluate both the reference and comparative interpolation on a grid spanning the measured temperatures. This grid ranges from 75 to 273 K, with steps of 0.1 K.

Subtracting the reference from the comparative measurement, given in Fig. 2a, we obtain Fig. 2b. This curve represents the left-hand side of Eq. 3. We know that the applied heat loads were 0 and 2 W in the reference and comparative measurement, respectively. Therefore, according to Eq. 4, the heat capacity is 2 W divided by the curve in Fig. 2b, resulting in the curve presented in Fig. 3. In order to use this result in our model, we smooth the calculated heat capacity and use a linear interpolation fit between the points

To determine the conductance K according to Eq. 8, we need the conductive heat flow $\dot{Q}_{0,1}$. This is obtained from Eq. 9. We use the smoothed C curve as in Fig. 3 to evaluate the heat capacity, which we multiply by the temperature-time derivative. The applied heat load $\dot{Q}_{\mathrm{app}}$ is also known. As stated above, the parasitic heat load $\dot{Q}_{\mathrm{par}}$ has to be estimated. As the aim of this paper is to demonstrate the method, we take $\dot{Q}_{\mathrm{par}} = 0$ W; i.e., the parasitic heat load is included in the conductive heat load $\dot{Q}_{0,1}$ and thus also in the conductance K. The conductive heat flows that we obtain in the reference and comparative cases by filling in Eq. 9 are given in Fig. 4a. Similar to the heat capacity C, the aim is to arrive at a smooth curve. We therefore fitted a single quadratic polynomial through the $\dot{Q}_{0,1}$ data

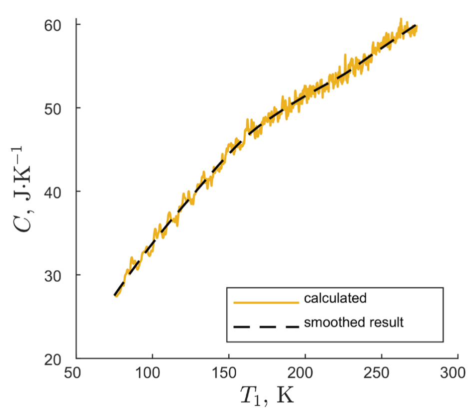

Figure 3. Heat capacity as calculated from Eq. 4, based on the difference in temperature-time derivatives as shown in Fig. 2b.

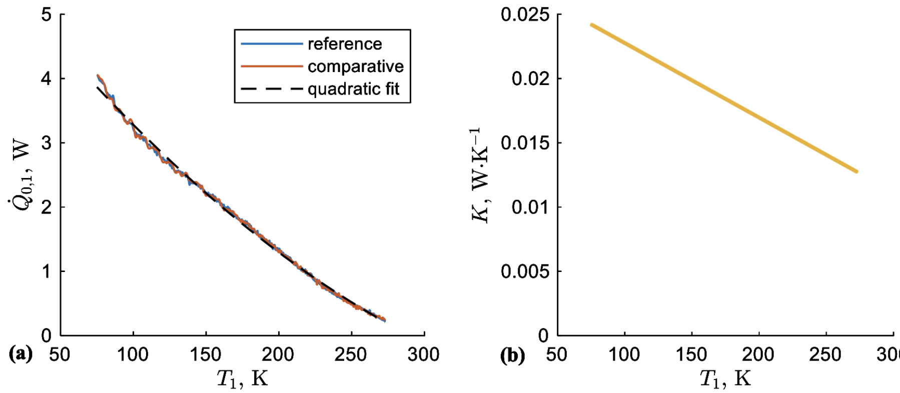

Figure 4. (a) Conductive heat flow $\dot{Q}_{0,1}$ in the illustrative case of the Thales pulse tube cooler resulting from Eq. 9 for the reference and comparative data and a quadratic fit to the total data set; (b) Conductance as a function of T_1 based on Eq. 8 resulting from the quadratic fit in (a).

of both cases. By taking the derivative of this quadratic expression with respect to temperature T_1 as indicated in Eq. 8, we arrive at a linear expression for the conductance as shown in Fig. 4b. Obviously, if the heat flow $\dot{Q}_{0,1}$ cannot be fitted to a simple differentiable function, a numerical method will need to be applied for determining the conductance.

At this point, the parameters of the thermodynamic model of this specific cooler have been determined (heat capacity in Fig. 3 and conductance in Fig. 4b). When this cooler is placed in a system, we can simulate the warm-up behavior of that system after the cooler has been switched off.

For that purpose, as indicated above, we will need to know the heat loads applied to the cooler in that system (other than the conductive heat flow through the cooler). Next, the temperature step ΔT needs to be defined with which the warming-up process of the cold tip is to be simulated. At each temperature, starting from an initial tip temperature, the conductive heat load $\dot{Q}_{0,1}$ is determined using the conductance K. The other heat loads on the tip are added and the time needed for a temperature increase by ΔT is determined based on the heat capacity and the total heat load. Since the objective of this section is merely to demonstrate our method, we have simulated the warming-up of the setup in which the warm-up curves of Fig. 1 and Fig. 2 were measured. A temperature step of 0.1 K is used in these simulations. The results of the simulated warm-ups are presented in Fig. 5. These results match the data very well, which is not surprising since the model parameters are determined using the same experimental data. Nevertheless, it serves the purpose of illustrating our method and model.

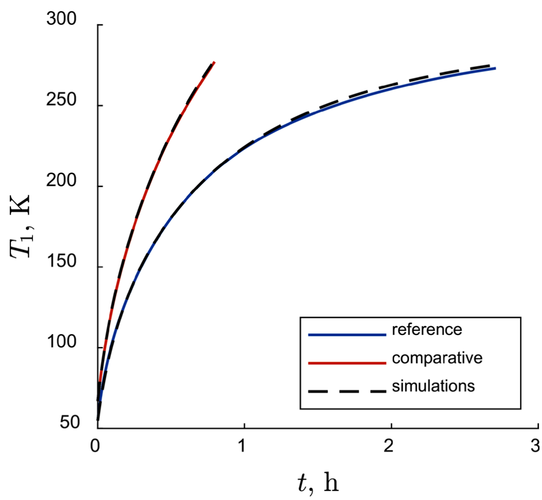

Figure 5. Warm-up curves of Fig. 1a and simulations using the thermodynamic model with heat capacity of Fig. 3 and conductance of Fig. 4b.

CONCLUSION

We have presented a simple method for predicting the thermodynamic behavior of a cryocooler as it is switched off. This was done with an RC model, using thermal capacitance for the stages and conductance for the thermal link between the stages. These properties are experimentally determined by letting the cryocooler warm up from a cold state, with different heat loads applied to the cold stage. The method was demonstrated on an LTP9310 single-stage pulse tube cryocooler. A clear advantage of the presented method is that the dimensions and materials, nor the internal composition of the cooler have to be known or estimated.

This characterization method is relevant for systems in which the dynamics during the time the cryocooler is in its off-state are of importance. Applications include vibration-sensitive systems in which the cooler cannot remain active during operation of the device that is cooled. One such system is the thermal noise measurement setup for the mirrors of the Einstein Telescope, that we are currently designing.

The RC model can be extended with a second C and K to model a two-stage cryocooler. This requires a critical review of the equations and the method itself, since this introduces variables that depend on the temperatures of both stages. Treating equations as a function of a single variable (the cold stage temperature) is therefore no longer possible. This work is in progress.

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REFERENCE

1. Hild, S. et al., “Sensitivity studies for third-generation gravitational wave observatories,” *Classical and Quantum Gravity*, Vol. 28, (2011).
2. *Einstein Telescope*, et-gw.eu, (2008).
3. Jagga, S., Holland, H. J., Vanapalli, S., “Systematic approach to determine the transient cooling power and heat leak of a commercial pulse tube cryocooler,” *Cryogenics*, Vol. 113, (2021).